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# Piston-Ring Friction in High-Speed Engines

By LOUIS ILLMER,[1] CORTLAND, N. Y.

**The author points out that the recent trend toward higher rotative speeds in automotive engines has revealed an accompanying rise in frictional losses caused by an accumulative pressure build-up in the annular pocket under the first piston ring. Reference is made to various data which can be used to determine with reasonable accuracy friction losses of a sliding piston ring. These data are treated analytically and coordinated to establish criteria for normal frictional ring drag at different rotative speeds and pocket pressures. A primary objective of the paper is to evaluate the piston-ring friction of a heavily loaded engine running at its highest speed.**

THE recent trend toward higher rotative speeds in automotive engines has revealed an accompanying sharp rise in frictional losses. Abnormal piston-ring drag is primarily responsible for such losses since piston rings tend to behave in a characteristic accumulative or self-loading manner that has not been clearly identified. When segregated, this frictional drag is found to increase progressively with augmented piston speed, and if it were allowed to reach inordinate proportions, the optimum mechanical efficiency could not be achieved.

This aspect assumes particular importance when applied to multicylinder engines of comparatively small bore in which the ratio of combined cylinder contacting surface of the sealing rings to the piston area is unduly large. For instance, when the speed of such an engine is raised from 1000 to 4000 rpm, the loss by ring friction materially increases the initial ring drag per unit of piston area.

Such peculiar performance evidently is caused by a relatively high pressure that builds up accumulatively in the annular pocket chamber under the first or breaker ring whenever the piston of a four-stroke engine piston is operated in excess of 1000 rpm. Sufficient test data have become available from scattered sources to enable the engineer to determine with reasonable accuracy the character of friction that prevails under a sliding piston ring. These experimental findings have been treated analytically and coordinated to establish a criterion for normal frictional ring drag at different rotative speeds and pocket pressures.

A primary objective of the present investigation is to evaluate the piston-ring friction for a heavily loaded engine while running at its highest speed. Actual determinations for the resulting pocket pressures are inherently difficult to obtain experimentally, hence values from tests at lower speeds must be projected into such an unexplored region. The equational method herein pursued forecasts the expectations under such extreme load conditions, although such treatment necessarily is speculative to some extent. More consistent results in separating internal losses are attained when the engine is motor-driven rather than run under its own power.

It will presently be shown that below a certain critical speed, the fluid charge entrapped within the first ring pocket closely follows the cyclic pressure change that occurs in the combustion chamber. The pocket pressure then fluctuates in unison with the intermittently applied piston pressure and reaches the maximum cylinder pressure. A commensurate momentary peak pressure under the first or breaker ring is necessary to provide for a reasonably tight piston seal. In a four-stroke engine, the usual lateral gap or normal groove play of a snugly fitted piston ring still affords sufficient port area through which the periodic cylinder pressures can be equalized readily, provided the rate of repetition is kept below a certain critical value.

In a four-stroke engine, the low-pressure exhaust and suction strokes bring about a more intensive venting of the first ring pocket and thus effects a corresponding reduction in the average ring thrust against the cylinder wall when running beyond such a critical speed. In the case of a two-stroke engine, the cited critical speed is lowered to about one half the critical speed of a four-stroke engine. When the intermittent cylinder pressure is applied in more rapid succession, the pressure in the breaker-ring pocket is no longer directly responsive to the full range of cylinder pressure and thus brings about a sustained ring loading between pressure applications.

Piston-ring drag is largely fixed by the mean integrated pressure maintained behind the respective rings. When such an average unit pressure is carried beyond proper limits under any one ring, the underlying law of friction is changed and an appropriate friction coefficient must be determined for each different circumstance.

A perfectly tight piston ring operating with substantially no leakage or blow-by would involve excessive frictional losses. While a relatively small portion of the cylinder charge is usually allowed to blow over the rings, the major portion of such blow-by should flow successively under the series of rings in labyrinth fashion and be trapped within the respective pockets. It is the first control ring that is subjected to the most severe duty. The remaining rings encounter a commensurate but progressively lesser range of pressure fluctuation. In a three-ring system, the working fluid entrapped in the last ring pocket is preferably reduced and kept at a comparatively low pressure. The charge under the medial ring is likely to be subjected to a fluctuating pressure intermediate to which the first and last rings are subjected. When the first and second rings are side-fitted snugly into their grooves, the pressure drop occurring over each of these two rings may be taken as approximately equal in head magnitude.

At slower speeds, the pressure in the first ring pocket may be expected to drop to that of the atmosphere when the exhaust valve is opened. However, at speeds materially higher than the

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critical rate of pressure application, the normal ring clearance or lateral gap becomes inadequate in vent capacity to release its pocket pressure completely. Hence, a fresh charge will be forced under the first ring before its previous pocket charge has dispelled itself and become entirely dissipated.

Consequently, an accumulative loading effect is initiated under the first ring that may reach a sustained average pressure far higher than the mean cycle pressure. A progressive pressure of this kind acts to augment the frictional ring drag because the fluctuating pressure in the pocket beneath the first ring is upheld and no longer drops to the low initial-pressure level which prevails under slower running conditions. The discussed accumulative pressure is not confined to engine pistons but likewise occurs in motor-driven compressors of the high-speed reciprocative type.

That such building up of ring pressure is actually established and assumes vital importance to high-speed-engine designers, may be traced by the following procedure.

It is advisable to first lay down a rough basis for determining normal blow-by loss or outward leakage past the piston rings that is deemed allowable with increased rotative speed. Piston blow-by is found to bear a close relationship to the accumulative ring pressure. To this end, reference can be made to some tests by H. M. Bramberry[2] of a six-cylinder $3^1/_4 \times 5$-in. four-stroke engine, in which the discharged leakage was measured in a cooled state through a suitable gas meter. The blow-by varies with the mean cycle pressure acting behind the piston and, while not definitely reported, its estimated value has been taken at 35 lb per sq in. The following equation is based upon medial-test determinations for such an engine and is deduced on the assumption that the anticipated blow-by increases with the square of the cylinder bore. Thus

$$B = \frac{D_c^2}{12} \sqrt{(NP_{cyc})}, \text{ approximately} \quad [1]$$

where $B$ = typical piston blow-by per cylinder for a four-cycle engine, cu in. per min; $D_c$ = cylinder diameter, in.; $N$ = speed, rpm; and $P_{cyc}$ = basic mean pressure prevailing behind the piston as taken over one entire cyclic period, lb per sq in.

Other data have been checked against Equation [1] with fair corroboration. Piston blow-by changes with ring-fit, lubrication, and bore conditions; hence its actual value is likely to range rather widely between $B/2$ and $2B$, although modern practice strives to hold such loss within restricted limits. Tests conducted by R. R. Teetor and H. M. Bramberry[3] show the blow-by expectation under different engine loads. When Equation [1] is divided by $N$ to ascertain the blow-by per stroke, this varies as $1/\sqrt{N}$ and reveals a reduced unit leakage with increased running speed.

For a four-stroke engine, an empirical value for the factor $P_{cyc}$ may be taken as

$$P_{cyc} = \frac{1}{2}\left(\frac{P_{com}}{4} + \frac{\text{mep}}{2}\right) \quad [2]$$

where mep = indicated mean effective pressure of the engine card, lb per sq in.; $P_{com} = 32(R - 1.7)$, approximately = maximum compression pressure in a four-stroke engine, lb per sq in., gage; and $R$ = ratio of compression when ranging between 3 to 8.

The equation factor $^1/_2$ allows for the two idle strokes, while $P_{com}/4$ takes account of the mean compression pressure when limited to a four-stroke explosive engine.

B. T. Robinson and A. R. Ford[4] succeeded in measuring the ring-pocket pressure for a $6^1/_2 \times 11$-in. engine while running up to 290 rpm under full-load conditions. These slow-speed determinations were made by maintaining counterbalanced pressures upon opposite sides of a small differential disk. When using a side fit or clearance gap of 0.002 in., the first or breaker ring was found to follow the changing cylinder pressure closely. The mean pressure under the second ring dropped to about one half that in the pocket under the first ring, while the third ring showed an abnormally high cyclic-pressure behavior, quite similar to that prevailing in the second ring pocket.

When testing a worn piston ring with a 0.010-in. gap, the second ring revealed a raised pressure fluctuation almost identical with that prevailing in the pocket under the first ring. Had these worn rings been fitted more closely, the pressure in the pocket under the third ring might have been reduced to a much lower value. However, this evidence clearly establishes a mean-pressure expectation under the first ring that is substantially equal to $P_{cyc}$, provided the speed conditions are not allowed to induce a progressive building up of such a base pressure.

This test further demonstrates that the use of loosely fitted piston rings materially increases the mean pressure under the second and third rings. This leads to a correspondingly higher frictional drag. The inherent tendency to raise all of the ring pockets to a common pressure level, makes it apparent that piston-ring friction is likely to become magnified after the master ring has been subjected to excessive groove wear.

Current practice in lateral fits for a concentric snap ring into a piston groove, may be taken as follows:

$$W = \text{about } D_c/25 \text{ for passenger car engines} \quad [3]$$

$$W = \text{about } D_c/20 \text{ for truck and bus engines} \quad [3a]$$

$$T = \text{about } D_c/30 \quad [4]$$

$$G = \text{about } \sqrt{W}/300 \text{ to } \sqrt{W}/400 \quad [5]$$

where $W$ = width of piston-ring groove, in.; $T$ = radial thickness for a concentric cast-iron snap ring, in.; and $G$ = normal groove gap or side-fit allowance for a compression ring, in.

The prescribed thickness $T$ corresponds to an initial expansive tension $P_0$ of about $3^1/_2$ to 4 lb per sq in. of bore contacting surface. The stipulated side fit $G$ still allows for a correspondingly small lateral movement of the ring within the groove confines since a certain amount of such play is needed to charge properly the underlying pocket against undue ring leakage. The use of but two compression rings in separate grooves is not unusual in present-day high-speed automotive and airplane engines. In order to reduce piston drag to a minimum under heavy engine loads, the ring width $W$ is generally held to $^1/_8$ in. even for bore sizes up to 6 in. and over, but such an extremely narrow snap ring is intrinsically frail.

Referring now to the character of friction coefficient that applies to a flat piston-ring surface, this is found to be of a rather low order because the rubbing surface of a heavily loaded ring when subjected to a high pocket gas pressure is not adequately sustained by an unbroken film in the manner of a perfectly oil-borne journal.

For purposes of comparison, reference is made to an earlier analytical survey[5] made by the author on unbroken film lubrica-

[2] "Pistons and Oil-Trapping Rings for Maintaining an Oil Seal," by H. M. Bramberry, Transactions, Society of Automotive Engineers, vol. 23, 1928, p. 418.

[3] "Piston Ring Progress," by R. R. Teetor and H. M. Bramberry, Transactions, Society of Automotive Engineers, vol. 27, 1932, p. 323.

[4] "An Investigation of Piston Ring Groove Pressures," by B. T. Robinson and A. R. Ford, *Diesel Power*, vol. 12, October, 1934, p. 548.

[5] "Friction Coefficient Research," by L. Illmer, *Society of Automotive Engineers Journal*, vol. 26, January, 1930, p. 67.

tion in rotating journals and for which the frictional resistance depends primarily upon fluid shear. As applied to present needs, the results of such research may be summarized by the formula

$$f = \frac{C_1C_2}{7.5}\,\frac{n^3\sqrt[4]{n}}{(P_r)^n}\,\sqrt{\mu V^{2/3}} \qquad [6]$$

where $f$ = normal coefficient of friction for an unbroken oil film
$C_1$ = type constant which is equal to about 2 for flat surfaces running on relatively long guideways
$C_2$ = circumstance constant the numerical value of which is never less than unity even under flooded lubrication, and which constant is dependent upon the rate of oil supply, workmanship and similar operating conditions
$P_r$ = about $P_{cyc} + P_0$ = integrated or mean ring pressure acting radially against the cylinder, lb per sq in. of bore contacting surface
$P_0$ = initial snap tension of an expansible piston ring, lb per sq in.
$n$ = pressure exponent, the maximum for which may reach unity for rotating journals under most favorable lubrication and radial-fit conditions
$\mu$ = absolute viscosity of the lubricant in poises, cgs units
$V$ = mean rubbing velocity of the piston ring or piston speed fpm

Certain constants for Equation [6] may be evaluated from the piston-ring tests reported by T. E. Stanton[6] for a very low rubbing velocity. These determinations were obtained from tests conducted with special apparatus comprising a pair of interconnected pistons which were slowly reciprocated by a heavy pendulum.

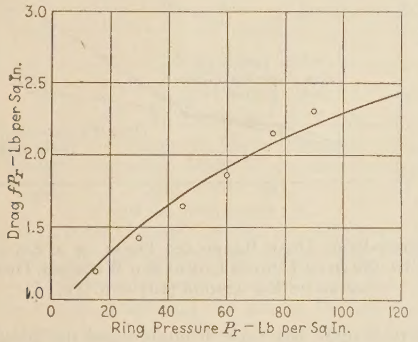


FIG. 1 PISTON-RING DRAG BASED ON STANTON'S TESTS
(Values of $fP_r$ used in plotting the curve are found by Equation [7] with $\mu$ = 1/2, approximately.)

The retardation rate of its oscillatory movement served to define the piston drag while a known air pressure ranging from 15 to 90 lb was maintained behind the several rings. Fig. 1 shows the results of the Stanton tests as developed for present purposes. Taking the mean piston speed at $V$ = 10 fpm the expression for the unit drag, representative of these findings at an oil temperature of 212 F, is

$$P_r f = P_r \frac{\sqrt{(\mu V^{2/3})}}{3.1(P_r)^{2/3}}, \text{ approximately} \qquad [7]$$

A significant deduction lies in fixing upon the exponent $n$ at 2/3. As based upon Equation [6], the numerical value of $C_1C_2$ becomes equal to 9.1. The resulting high circumstance constant $C_2$ of about 4.5 may be ascribed to a lack of uniform oil distribution around the entire cylinder bore. For faster piston speeds, the increased oil splash facilitates lubrication and hence would make for a better showing of the $C_2$ factor.

[6] See abstract in *Automotive Industries*, Feb. 1, 1925, p. 276.

More enlightening piston-ring tests have been reported by R. L. Streeter and L. C. Lichty[7] in which a White truck engine was used. By successively motor-driving this four-cylinder engine after individually removing certain of its elements in accordance with G. B. Carson's method, the proportionate frictional loss

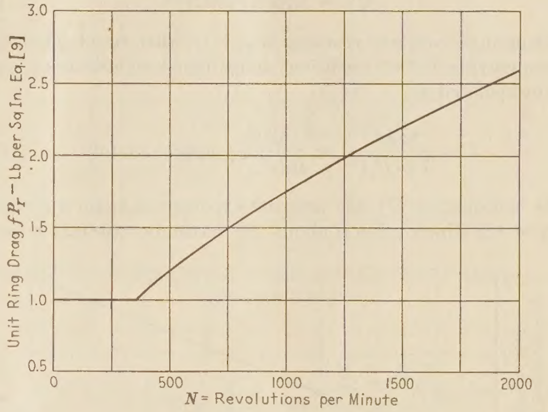


FIG. 2 CUMULATIVE PISTON-RING DRAG BASED UPON TESTS OF A WHITE TRUCK ENGINE

corresponding to each such element may be evaluated. As an inherent limitation, this method is confined to light-load running conditions in which the factor $P_{cyc}$ of Equation [1] remains at a relatively low value.

If this $4^1/_4 \times 5^3/_4$-in. four-stroke engine had a compression ratio $R$ = 4.3 and were running at 1600 rpm, it would have a rated power of about 54 bhp at 81 lb per sq in. brake mep. With the throttle wide open, the gross internal or friction-horsepower loss of such a four-cylinder engine when motor-driven, may be fixed at

$$\text{fhp} = 5.7\ (N/1000)^{19/16} \qquad [8]$$

Other nonfiring, motor-driven tests[8] show the same characteristic rise in frictional losses with increased speed. In the case of the White truck engine running at a jacket-water temperature of 180 F, the losses in the several rings have been broken down and separately appraised. The loss due to the first compression ring, to the second compression ring, and to the scraper ring and piston thrust is 44, 34, and 22 per cent, respectively, of the total ring-friction loss. The sum of these losses amounts to about 27 per cent of the gross friction horsepower as given by Equation [8].

In this particular low-speed test, the ratio of the total ring friction to the gross friction-horsepower loss remains substantially constant, irrespective of normal variations in the jacket-water temperature. The given test data is also inconclusive in regard to the effect of oil viscosity, which apparently exerts no marked influence except at lower running temperatures.

The present investigation is especially concerned with the behavior of the first or breaker ring. The tests under discussion afford sufficient data to establish the unit piston drag $fP_r$. At a

[7] "Internal Combustion Engines," by R. L. Streeter and L. C. Lichty, McGraw-Hill Book Company, New York, N. Y., fourth edition, 1933, p. 434.

[8] "Why Oil-Cooler for Engines?" by S. W. Sparrow, *Automotive Industries*, vol. 64, June 13, 1931, p. 916. This paper shows that the frictional losses in an eight-cylinder, $3^1/_2 \times 4^3/_8$-in. Studebaker engine rises at a relatively faster rate when fhp = 6.35 $(N/1000)^{15/8}$.

ring width $W = 1/4$ in., or an entire first-ring area of about 13.35 sq in., such unit-drag values may be found by taking the proportionate friction-horsepower loss assigned to the first ring and dividing this by its corresponding piston speed. The results are shown in Fig. 2. They indicate clearly a striking accumulative rise in frictional drag with increased rotative speed. Accordingly, the ring drag per unit of bore contact area becomes approximately

$$fP_r = 1.75(N/1000)^{9/16} \qquad [9]$$

Assuming a constant viscosity at $\mu = 1/5$ when running hot, the accompanying friction coefficient under light-load conditions, may be interpolated as

$$f = \frac{\sqrt{\mu V^{2/3}}}{7.2\,(P_r)^{2/3}} = \frac{\sqrt[3]{V}}{16(P_r)^{2/3}}, \text{ approximately} \qquad [10]$$

As in Equation [7], the pressure exponent is again appraised at $n = 2/3$, which value is about the maximum obtainable for a

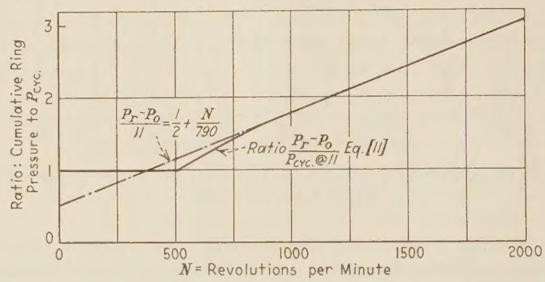


Fig. 3 Ratio of Cumulative Pressures Under Piston Rings to the Mean Cyclic Pressure in the Cylinder at Various Speeds of a White Truck Engine

flat rubbing surface and implies a substantially unbroken film formation while Equation [10] remains operative. In accordance with Equation [6], $C_1C_2$ becomes equal to about 3.9 and shows a decided improvement in the circumstance constant. In this particular test, the ring rubbing surface was provided with an inserted narrow bronze bearing strip of which the beneficial effect would probably become more pronounced after the radial loading $P_n$ is no longer fully oil-borne. Furthermore, the test in question did not extend the upper speed range sufficiently to show an anticipated change in frictional behavior that presently will be pointed out.

By substituting the given value for $f$ into Equation [9], the following relationship may be deduced which is applicable to light-load operating conditions at an appraised value of $P_{cyc} = 11$ lb: When $P_r > P_{cyc}$, then $P_r = 1/5(N)^{11/16}$, or approximately

$$P_r = P_0 + \left(\frac{N^{11/16}}{5} - P_0\right)\sqrt{\frac{P_{cyc}}{11}} \qquad [11]$$

The factor $P_{cyc}$ of Equation [11] has been empirically taken on a square-root basis. This speed factor has been similarly treated in Equation [1]. It will be observed that the radial pressure $P_r$ between the ring and bore surfaces rises at a relatively fast rate with increased speed. For a motor-driven White engine, the value of $P_{cyc} + P_0$ has been estimated at about 15 lb under nonfiring conditions. Fig. 3 has been drawn from values obtained by dividing $(P_r - P_0)$ by its initial slow-speed value at $P_{cyc} = 11$ lb. The resulting quotient may be approximated on a simplified straight-line basis, thus

$$P_r = 11\left(\frac{1}{2} + \frac{N}{790}\right) + P_0 \qquad [12]$$

The divisor 790 of Equation [12] is found to change for a different $P_{cyc}$ value. The bracketed portion represents the sought for build-up factor that is appropriate under light-load conditions, and assumes a value of unity when $N$ is approximately 400 rpm. Below this speed, the ring-pocket pressure may be expected to remain substantially constant, while above such a critical speed, the pocket pressure starts to increase progressively at higher rotative speeds. Equation [12] has been defined in terms of $N$ since the cumulative ring pressure is dependent upon the frequency of application rather than piston speed.

Another promising procedure for studying the building up of ring pressure is afforded by motor-driving an engine with and without its cylinder heads. Certain tests reported by Austin M. Wolf[9] will now be analyzed to illustrate the results attained by this approach. Wolf tested a six-cylinder $3^1/_2 \times 4^1/_2$-in. Graham engine which was run beyond 3000 rpm. Each piston in this engine has two $1/8$-in. wide compression rings used in combination with one $3/16$-in. scraper ring.

At higher running speeds, the crankshaft bearings are primarily loaded by inertia of the reciprocating parts. Hence, any minor pressure increase due to compression would not materially alter the bearing losses. Accordingly, the difference in friction-horsepower losses obtained with the heads on and off, affords a fairly close measure of increased piston-ring friction, provided a suitable deduction is made for the expenditure in pump work while the heads are in place. The other internal losses are assumed to remain substantially identical at any given rotative speed.

The permissible flow drop through the manifold and valves

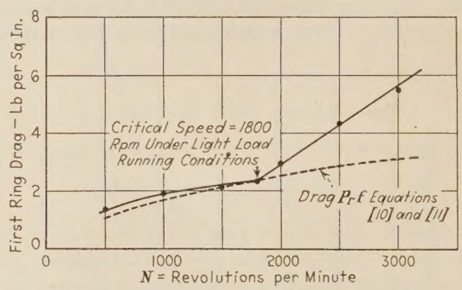


Fig. 4 Piston-Ring Drag Based on Tests of a Six-Cylinder, $3^1/_2 \times 4^1/_2$-In. Graham Engine Compared With the Drag Calculated by Equations [10] and [11]

depends in part upon the type of engine and its highest speed. For the Graham engine, the rated power has been taken as 65 bhp at 2600 rpm, and the corresponding pump-work loss is appraised at 3.3 ihp under full-load conditions. For other speeds, this fluid-flow loss has been apportioned in accordance with the detailed findings reported for the White truck engine.

As deduced for the White truck engine, 44 per cent of the total piston friction was assigned to the first ring. Because of the greater inertia pressure of the reciprocating parts and the resulting increased connecting-rod thrust in the higher-speed Graham engine, approximately 42 per cent of the total piston friction is credited to the first ring. The resulting breaker-ring drag is disclosed by the solid curve in Fig. 4. For comparison, the $fP_r$ drag corresponding to Equations [10] and [11] is represented by a dashed reference line which shows satisfactory agreement below 1800 rpm under light-load conditions. It is noteworthy that beyond this critical speed, the drag of the first ring in the Graham

[9] "A Critical Study of Car Design and Performance," by A. M. Wolf, *Society of Automotive Engineers Journal*, vol. 34–35, August, 1934, p. 11.

engine ceases to follow the projected equation values but clearly exhibits a progressive increase.

Fig. 5 presents the equivalent friction coefficient determinations for the Graham engine as based upon Equation [10] at $P_{cyc} = 15$ lb. The plotted friction coefficient undergoes a decided change in character when the rotative speed exceeds 1800 rpm and the accumulative pocket-pressure $P_r$ reaches a critical value of about 40 lb per sq in. beyond which incipient abrasion might be expected.

Such behavior is to be anticipated in accordance with another survey reported by the writer.[10] This was a study of intermediate or so-called boundary friction in which the rubbing surfaces are no longer separated by an unbroken oil film. In substance, these findings may be expressed in the following form as applied to a piston ring

$$P_rV^{2/3} = \frac{0.6}{C_2} P_n \sqrt[3]{H} = C_0 \ldots\ldots\ldots\ldots [13]$$

where $P_n$ = material constant whose value is about 4500 for close-grained cast iron on cast-iron rubbing surfaces

$H$ = cooling intensity factor or specific radiating capacity of the ring per deg F, ft-lb per min per sq in. of ring area in contact with the cylinder bore

$C_0 = P_rV^{2/3}$ = bearing constant for a continuously loaded piston ring when running on a ruptured oil-film surface without serious abrasion

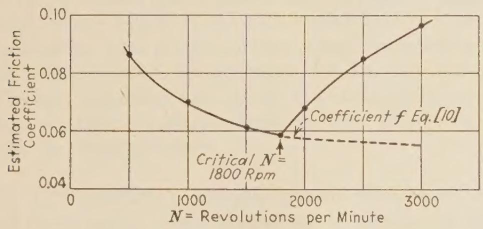


FIG. 5 FRICTION COEFFICIENTS FROM TESTS OF A SIX-CYLINDER GRAHAM ENGINE COMPARED WITH COEFFICIENTS CALCULATED BY EQUATION [10]

As long as $C_0$ is held below a certain critical value, the ring-friction coefficient may be expected to comply with Equation [10], but when $C_0$ is allowed to exceed its given value, then the sustaining oil film breaks down and the law of friction takes another course as shown in Fig. 6. From such a critical value onward, the friction coefficient no longer drops with $P_r$ but follows a different law, namely

$$f_{exc} = V^{2/3}/1900 \ldots\ldots\ldots\ldots\ldots\ldots [14]$$

This derived piston-ring value increases rapidly with the velocity factor for the reason that the accumulative pressure $P_r$ has herein been incorporated in terms of $V$. The value of $f_{exc}$ now rises with pressure instead of being inversely proportional to the pressure as in Equation [10]. It will be evident that a very considerable radiating capacity must be provided to properly cool a fast-moving ring drag of the given magnitude. As taken upon a unit-area basis, such a requirement may be roughly traced by the relation

$$Ht_h = f_{exc}P_rV = C_0(V/1900) \ldots\ldots\ldots\ldots [15]$$

where $t_h$ = mean temperature head available between the ring and the cylinder bore for cooling the ring, F.

[10] "High-Pressure Bearing Research," by L. Illmer, Trans. A.S.M.E., vol. 46, 1924, p. 883.

A certain amount of ring wear is encountered whenever $C_0$ is run much in excess of its critical base value of say about 5000 when $H = 40$. A well-lubricated piston ring then goes over into a stage where partial film breakdown has been initiated and some abrasion is to be expected. As based upon machine-tool bearing practice, excessive abrasion and probable seizing of such a piston ring is likely to occur whenever the $PV^{2/3}$ value exceeds three

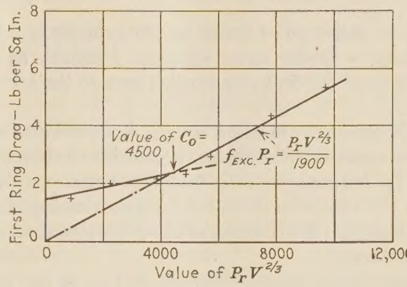


FIG. 6 PISTON-RING DRAG BASED ON TESTS OF A SIX-CYLINDER GRAHAM ENGINE

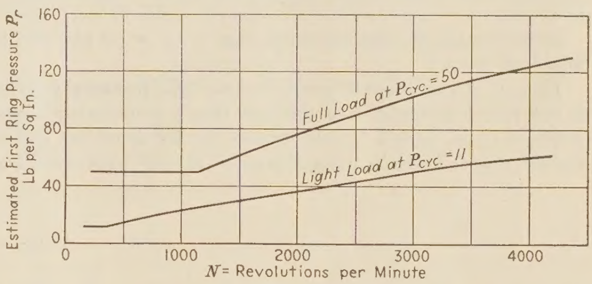


FIG. 7 PRESSURE UNDER THE PISTON RINGS OF A 3 × 4-IN. ENGINE AS CALCULATED BY EQUATION [11]

times its critical value or about $C_0 = 15{,}000$ at $H = 40$, which in turn corresponds to the minimum allowable safety factor.

In a recent test[11] of a $3^1/_8 \times 4^1/_4$-in. six-cylinder automobile engine in which each piston was equipped with three $^1/_8$-in. wide rings, a constant gas pressure of known value was maintained behind each piston while running. The data obtained in this test do not permit a close evaluation of the ring coefficient $f$. As interpolated for present purposes, this coefficient at 1200 rpm is approximately 0.06 at 20 lb gas pressure and slowly drops to 0.04 at 120 lb gas pressure, the value of the pressure exponent being reduced to about $n = {}^1/_4$. At 1850 rpm, the corresponding friction coefficient $f$ still further changes its character into a substantially constant value of approximately 0.05 and is evidently about to rise in accordance with Equation [14]. However, the maximum speed of 1850 rpm is hardly sufficient to bring about any marked accumulative rise in ring drag. Taylor[11] estimates his mean friction coefficient with increased gas pressure at about 0.06; he also points out that such rings do not ride upon a perfect oil film and finds the ring drag $fP_r$ to increase in a linear relationship with speed.

The safe ring pressure must be reduced sharply with increased speed. Taking $P_{cyc} = 50$ lb according to Equation [2], the foregoing may be applied to a fully loaded 3 × 4-in. engine running up to and beyond 3500 rpm. The resulting pernicious rise in $P_r$ as based upon Equation [11] is shown in Fig. 7. For comparative purposes, the light-load pocket pressures acting under the first ring are also shown in Fig. 7. This accumulative pocket

[11] "The Effect of Gas Pressure on Piston Friction," by M. P. Taylor, *Society of Automotive Engineers Journal*, vol. 38, May, 1936, p. 200.

pressure is shown to impose a decided extra braking action upon the engine, particularly under heavy high-speed loads.

Finally, by taking the bore contact area of the first ring in terms of piston area, a simple relationship may be deduced for the drag expectations of a loaded four-stroke engine. Thus

$$\frac{\text{hp of first-ring friction drag}}{\text{bhp per cylinder}} = \frac{4(f_{exc}P_r)}{\text{bmep}}K \quad \ldots [16]$$

where bhp = delivered or brake hp per cylinder of a four-stroke engine, bmep = brake mean effective pressure, lb per sq in., and $K$ = ratio of the first-ring contact area to the area of the piston head.

When all the rings are fitted snugly in accordance with Equation [5], the relative frictional loss ascribable to the several piston rings may be deduced from the foregoing equations and observations. In the case of a 3 × 4-in. engine running at 3500 rpm under full load, the distribution of the estimated drag may be appraised as follows:

Proportionate loss for the first ring = 1 = 60 per cent of such total loss.

Proportionate loss for the second ring = $^1/_2$ = 30 per cent of total loss.

Proportionate loss for the third ring = $^1/_6$ = 10 per cent of total loss.

Thus, $1/1^2/_3$ or about 60 per cent of the total frictional loss due to the piston is credited to the first ring. Considering a ring width of $^1/_8$ in. for a 3 × 4-in. engine, the frictional loss due to the first ring as found by using Equation [16] is $7^1/_2$ per cent when the brake mean effective pressure is 90 lb per sq in. The corresponding gross friction loss, including the connecting-rod thrust thereon, becomes equal to $7^1/_2/0.60$ = about 12.5 per cent of the rated brake horsepower of the engine when carrying a load of 11.3 bhp per cylinder. Such wastage of approximately 1.4 hp for each piston at $P_{cyc} = 50$ points to an unduly high power dissipation that should offer a fruitful field in which to improve engine efficiency further. Corroborative test figures have been deduced from the previously cited Wolf report[9] and Sparrow[12] pertaining to several similar engines running at maximum load.

Attention is also directed to the ring inertia effect which tends to seal the side of the ring and restrict the blow-by from flowing through its underlying pocket chamber. Even when the width $W$ is reduced to $^1/_8$ in. the maximum inertia pressure against the groove side wall may reach close to 20 lb per sq in. at highest speeds, which correspondingly obstructs the venting of a closely fitted ring gap $G$.

The foregoing analysis is thought to throw some light on the described build-up pressure behavior of piston rings and to stress the more essential operative factors that need to be considered in seeking a rational solution for the underlying lubrication problem. While the cited experimental data may not be wholly conclusive, the offered treatment presents appropriate guidance on which to predetermine piston-ring performance under heavy engine loads when running at high speeds.

[12] "Recent Developments in Main and Connecting-Rod Bearings," by S. W. Sparrow, *Society of Automotive Engineers Journal*, vol. 25, July, 1924, p. 229.

# Design and Operating Problems With Gas- and Oil-Fired Boilers for Stand-By Steam-Electric Stations

By V. F. ESTCOURT,[1] SAN FRANCISCO, CALIF.

**The problem of stand-by operation, although not new on the Pacific coast, is a comparatively new experience for large units operating at pressures above 400 lb and temperatures above 700 F. With combination gas and fuel-oil installations, there are more difficulties to overcome with fuel oil than with gas. The paper describes problems in the control of flame angle when burning fuel oil and developments in high-speed automatic combustion control which make possible the acceptance of emergency loads from 5 to 90 per cent of turbogenerator capacity as fast as the turbine governors can open the control valves. Various operating procedures which have been developed as a necessary routine to meet stand-by conditions are also described. The essential requirements for stand-by and base-load operation are compared. Descriptions are given of actual load-pickup tests on a 1400-lb, 750-F reheat plant and a 425-lb, 750-F plant, and the actual events which take place in the first few seconds of an emergency load pickup are analyzed from the standpoint of what should be provided for in the design of boiler storage capacity, and speed of combustion control. Statistics relating to these features are given for four plants on the Pacific Coast.**

THE EXTREMELY variable power demands on steam and hydroelectric power plants from one season to another have introduced the problem of stand-by operationin to the routine of several Pacific-coast plants. Wide fluctuations in the steam requirements from year to year are brought about largely by seasonal variations in the amount of water storage available. During the recent depression years, the general falling off in power consumption also had its effect on steam plants that would normally be operated on base load, because the reduced output made it possible to meet the system requirements almost entirely on hydro, even in seasons which normally would have required base-load operation of the steam plants. The influx of government hydroelectric power in California promises to force the steam plants once again into the uneconomical conditions of stand-by operation for much longer periods than would otherwise have been the case.

The relation between steam and hydro output for two large systems on the Pacific coast for the past five years is presented in Table 1. Load-duration curves for the same period are given in Fig. 1 for plant *A* of company No. 1. These statistics are intended merely to furnish a general but representative picture of the type of operating conditions into which the steam-generat-

TABLE 1 COMPARISON OF STEAM AND HYDRO GENERATION BY TWO TYPICAL UTILITIES ON THE PACIFIC COAST

| | Per cent of total gross generation | | | |
|---|---|---|---|---|
| | Company No. 1 | | Company No. 2 | |
| Year | Steam | Hydro | Steam | Hydro |
| 1931 | 29.1 | 70.9 | 53.6 | 46.4 |
| 1932 | 6.4 | 93.6 | 8.6 | 91.4 |
| 1933 | 4.2 | 95.8 | 12.6 | 87.4 |
| 1934 | 9.3 | 90.7 | 30.2 | 69.8 |
| 1935 | 3.1 | 96.9 | 13.5 | 86.5 |

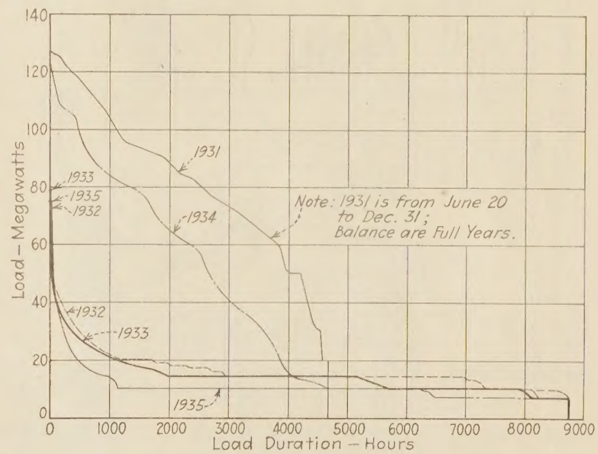


Fig. 1 Load-Duration Curves for Plant *A* of Company No. 1, Operating at 1400 Lb per Sq In.

ing stations are required to fit. It can be seen from the data that the steam plants of both systems have been required to operate on a stand-by basis for most of the time subsequent to the year 1931, although the year 1934 provided some base-load operation.

For the purposes of this discussion, the term "stand-by operation" is intended to apply to that type of service which requires the steam plant to operate at or near minimum load, but is prepared at all times to accept a large percentage of its total capacity as fast as the turbine governors will respond to sudden reductions in system speed, caused by transmission-line faults or complete separation of the steam and hydro systems for any cause whatsoever.

Although the problem of stand-by operation has been common among steam plants on the Pacific coast for many years, its

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Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until March 10, 1937, for publication at a later date. Discussion received after the closing date will be returned.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

application to large-capacity units operating at steam temperatures above 700 F and pressures above 400 lb is a comparatively new development. What has been accomplished is the result of considerable research and experimentation. The main objective has been to make the practically instantaneous acceptance of large blocks of load a matter of normal operating routine which can be accomplished either semiautomatically or fully automatically from a central control board without undue reliance upon the human factor and with practically the same degree of reliability as can be expected in the case of ordinary load changes.

The investigations which form the basis of this discussion were conducted in two different plants, the one designed for a pressure of 1400 lb per sq in., with initial and reheat steam temperatures of 750 F, and the other for a pressure of 425 lb per sq in. and 750 F steam temperature. Details of the major equipment in these two installations are given in Appendixes 1 and 2. Sectional elevations of the boilers are shown in Figs. 2 and 3. The essential problems in connection with stand-by operation will first be considered. Representative tests conducted in the above plants will be described later.

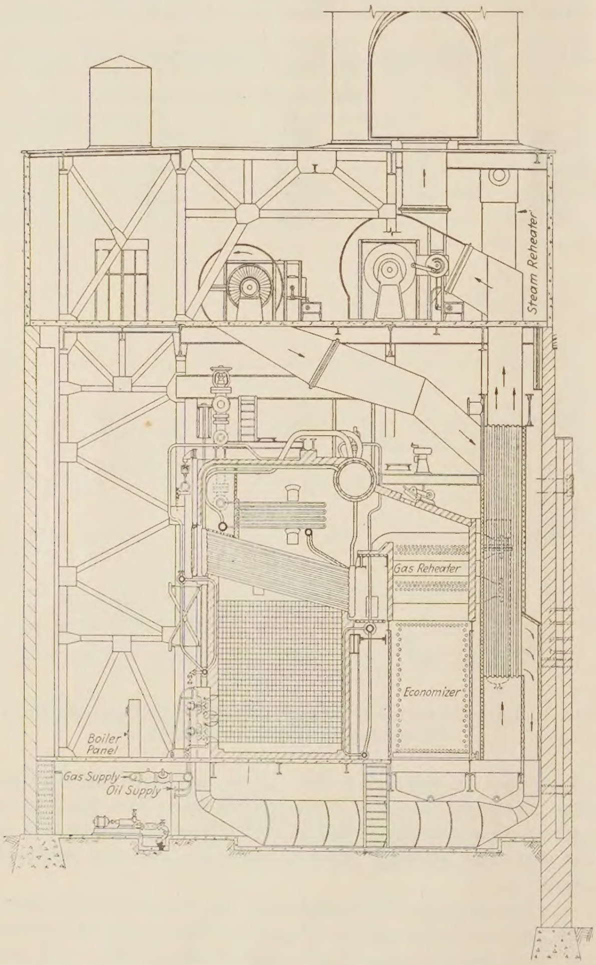


Fig. 2 Sectional Elevation of the 1400-Lb Pressure Boiler Used in Load-Pickup Tests

## Gas Versus Fuel Oil for Stand-By Operation

For stand-by operation, the kind of fuel burned enters very definitely into the picture. At the two plants under consideration, the boilers are equipped for burning either natural gas or fuel oil. The amount of operation with each kind of fuel is given in Table 2. Due to the absence of pumps, heaters, and other auxiliaries necessary for burning fuel oil, natural gas is much the more desirable of the two fuels. Gas burners do not foul in operation, thus eliminating the burner-cleaning problem. Natural-gas burners have a slightly wider operating range than do oil burners because of the fact that the minimum pressure at which they can be operated with safety and proper combustion corresponds to a lower load than is possible with fuel oil.

No measurable difference can be observed in the speed of response to changes in fuel rates for either gas or oil fuel. The limitations with oil fuel are due to the difficulty in maintaining the correct flame angle and proper atomization with standard oil-burning equipment at low loads. Due to this fact, it becomes

TABLE 2 COMPARISON OF GAS AND OIL-FUEL OPERATION FOR TWO LARGE ELECTRIC GENERATING STATIONS ON THE PACIFIC COAST

| Year | Boiler hours per cent of total Gas | Boiler hours per cent of total Oil | Fuel burned per cent of total Gas | Fuel burned per cent of total Oil |
|---|---|---|---|---|
| | | Plant *A* | | |
| 1931 | Not available | | 98.0 | 2.0 |
| 1932 | Not available | | 93.8 | 6.2 |
| 1933 | 86.4 | 13.6 | 86.0 | 14.0 |
| 1934 | 93.5 | 6.5 | 97.0 | 3.0 |
| 1935 | 74.4 | 25.6 | 74.3 | 25.7 |
| | | Plant *B* | | |
| 1931 | 98.2 | 1.8 | 98.3 | 1.7 |
| 1932 | 99.6 | 0.4 | 99.5 | 0.5 |
| 1933 | 100.0 | 0.0 | 100.0 | 0.0 |
| 1934 | 99.9 | 0.1 | 99.9 | 0.1 |
| 1935 | 99.9 | 0.1 | 99.9 | 0.1 |

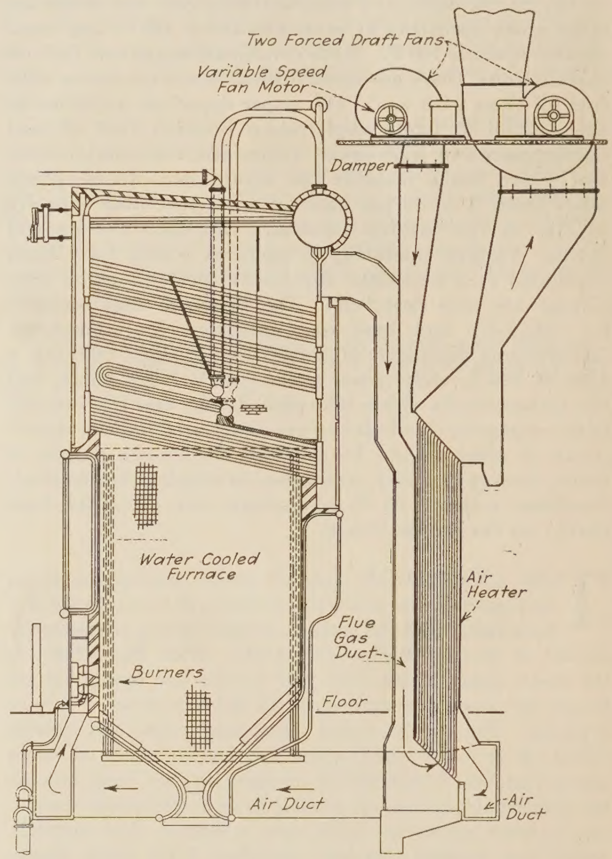


Fig. 3 Sectional Elevation of 425-Lb Pressure Boiler Used in the Load-Pickup Tests

necessary, as the load is reduced, to put out some of the burners in order to prevent the oil pressure at the burner orifice from falling below some predetermined value which has been found to be the minimum at which atomization is satisfactory for extended periods of time.

The type of oil burner in successful operation in several central stations on the Pacific coast is the so-called "wide-range mechanical atomizer," a cross section of which is shown in Fig. 4.

The oil passes through tangential slots into a small conical chamber and thence through the burner orifice at the apex of the cone. At the rear of the chamber, that is, at the base of the cone, there are a number of holes through which the oil

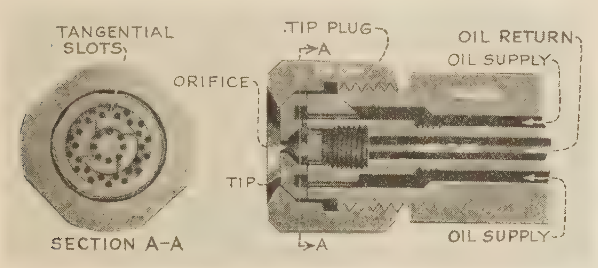


Fig. 4 Section Through Wide-Range Mechanical Atomizer Used in Connection With the Installation Shown in Fig. 5

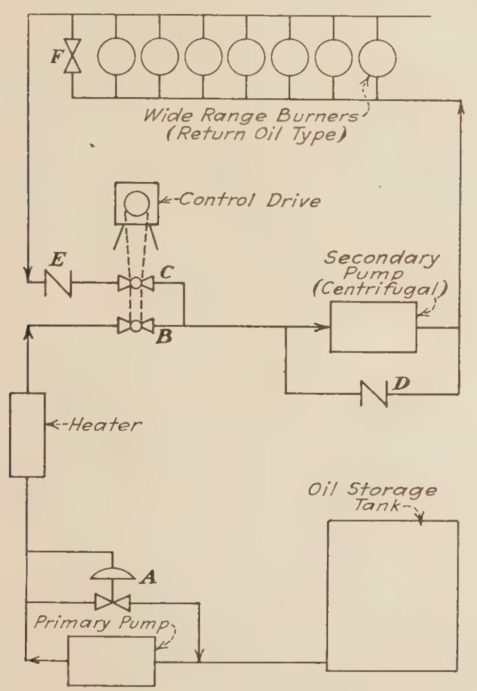


Fig. 5 Arrangement of Fuel-Oil Control System for Wide-Range Operation

may flow into a return pipe leading back to some point in the fuel-oil supply system. By regulating the flow of oil through the return line, the pressure at the burner tip, and consequently the oil discharge into the furnace, may be varied. This regulation is accomplished by means of a valve in the return line, the supply pressure to the burners being held at some constant value, usually around 200 lb. Due to this fact, the amount of oil passing through the tangential slots increases as the tip pressure is reduced in this manner, so that there is actually more oil in circulation at low rates of oil discharge into the furnace than at high rates. Since the flame angle is a function of the velocity of oil flowing through the tangential slots, and since the velocity through these slots is a function of the pressure drop across them, it can readily be seen that a reduction in return-oil pressure, with supply pressure held constant, will reduce the oil discharged by the burner, increase the velocity of oil flowing through the tangential slots and increase the width of flame angle. At very light loads, the flame angle becomes objectionably wide and among other things direct impingement upon the furnace floor results. If this condition is permitted to continue for a number of hours, carbon will accumulate on the furnace floor and ultimately cause serious obstruction to the path of the flame.

Thus, for very wide operating ranges, it is desirable to control the flame angle to meet the particular conditions. One method which has been devised for this purpose is a valve, installed in the supply line and controlled automatically by means of the differential pressure, between the supply and return lines to maintain this differential at some constant value. However, in order to meet the requirements of operating simplicity and reliability, the arrangement shown in Fig. 5 was considered preferable for the installation under discussion. While the initial cost may be slightly higher, from the operator's viewpoint it is much simpler, more rugged and inherently more flexible. It leaves the fuel supply completely under the control of the operator by means of a simple mechanical drive without any relief valves, differential-control valves or other automatic devices interposed. Such characteristics in the design cannot be emphasized too greatly in the interests of the high standard of reliability expected of stand-by service. The arrangement just described also makes possible the incorporation of any desired flame-angle characteristics over the load range by simply changing the relationship between the supply and return oil valves, this being accomplished by a change in drive-sprocket size or adjustment of lever-arm ratios as the case may be. The general characteristics are of course obtained by the original valve-port design, the final adjustments being made by the method just described.

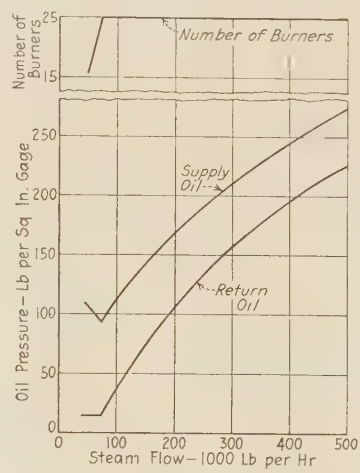


Fig. 6 Supply- and Return-Oil Pressure Characteristics Obtained With Fuel-Oil Control Installation Shown in Fig. 5

As a concrete example, the 1400-lb boilers on which this type of oil-burning equipment was installed have a maximum steam-generating capacity of 500,000 lb per hr. With the standard arrangement for the control of return-oil pressure, the minimum steam flow for satisfactory continuous operation with all burners

in service was approximately 130,000 lb per hr as compared with 30,000 lb per hr for gas fuel. However, after redesigning the job to conform to the layout in Fig. 5, a minimum load of 65,000 lb per hr could be carried for indefinite periods of time without getting into difficulties with carbon deposits. The actual supply and return-oil pressure characteristics for this particular installation are shown in Fig. 6.

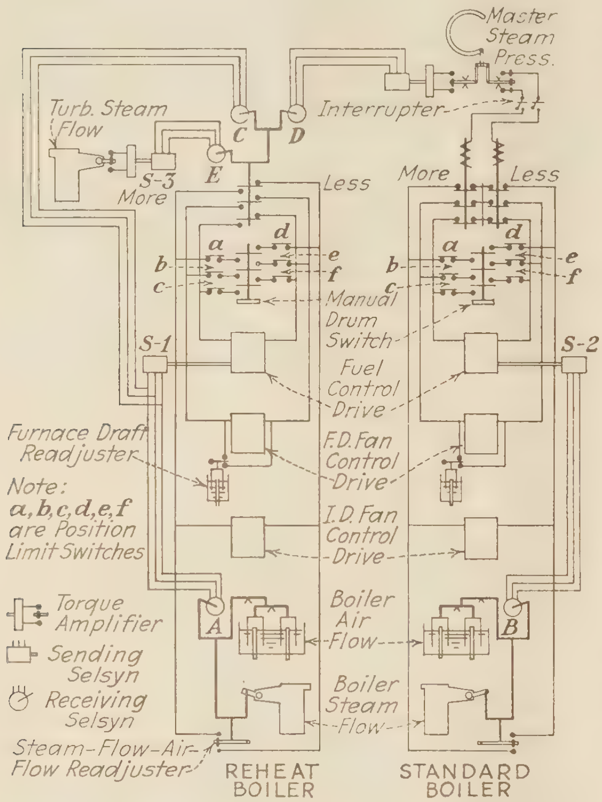


Fig. 7 High-Speed Combustion-Control System Used With 1400-Lb Pressure Installation

It is entirely possible that still lower loads may be obtainable as a result of further experimentation, but the data presented herewith represent what has been accomplished in this particular plant up to the present time. It should be borne in mind that there is no precise method of evaluating the minimum oil pressures below which satisfactory operation is no longer possible. The figures just given are based upon a conservative opinion as to what is a satisfactory operating condition in the furnace. It is entirely possible to operate the present installation down to the same minimum load with all burners in service as is obtainable with gas fuel, but it would not be considered feasible to carry these loads indefinitely for the reasons already explained.

## Combustion and Other Control Problems

For base-load plants, the combustion-control equipment has a relatively simple job to perform. It is required to operate satisfactorily over a load range of from possibly 50 to 100 per cent of capacity, to control small load changes at comparatively slow rates of speed and to be stable under steady load conditions.

On the other hand, the stand-by plant throws unusual demands upon the combustion-control system. In the first place, it is required to perform satisfactorily over a load range of from less than 5 per cent to 100 per cent of capacity. It must be capable of controlling small load changes at comparatively slow rates of speed, it must be stable during steady load conditions, and must be capable of handling at high speed sudden load changes of considerable magnitude. Roughly speaking, it must be capable of a control speed from three to five times faster than is necessary for base-load operation without sacrificing any of the characteristics necessary for the latter type of operation.

In Fig. 7 is shown an elementary diagram of one type of combustion control which has proved successful for stand-by operation. It is in use in connection with the 1400-lb installation.

It comprises two distinct systems, one semiautomatic and one completely automatic. The semiautomatic system consists of a manually operated drum controller mounted on the main control panel for each of the three boilers. By rotating this controller to the "more" position, a continuous impulse is sent to the fuel and fan controls which causes them to open up at rates of speed designed to preserve a predetermined relation as between air flow and fuel flow. The relation maintained between the control speeds of forced-draft and induced-draft fans is also such that the furnace draft is balanced at all times. If the drum switch is left in the "more" position, the fuel and air flow will increase until the fuel, forced-draft and induced-draft fan controls are arrested by the position-limit mercoid switches *a*, *b*, and *c* respectively. The limits are adjusted so that the vanes of the induced-draft fan are wide open, the forced-draft vanes are at a position which produces a balanced furnace draft, and the fuel valve is at a position which corresponds to approximately 10 per cent below the maximum steady-load fuel flow that is possible with correct combustion for the existing air flow. As soon as the drum switch is returned to the "off" position, the furnace-draft and steam-flow-air-flow readjusters come into operation and make any minor readjustments necessary.

The arrangement just described was installed shortly after the early investigations into stand-by operation were completed several years ago. It has proved very satisfactory in performing the particular functions for which it was designed. There are, however, other operating conditions which need to be considered. In the first place, it can be seen from the layout in Fig. 8, that all of the steam generated by either reheat boiler must return through the convection reheater of the same boiler, otherwise unbalanced reheat temperatures may result. Therefore, there is a preferred ratio of loading for the three boilers which will produce the best reheat characteristics. During load changes it is not easy to maintain these ratios without the aid of automatic control, and there is also the variable human factor. The latter also introduces an unknown element into the problem of handling emergency load pickups. It is all important that the delay in getting the controls started should be a minimum, particularly under certain conditions of operation, such as when there is curtailment of normal boiler capacity or when burning oil fuel, due to the extra time required to light the burners that are out of service at light loads.

In view of the aforementioned conditions, and also since nearly 90 per cent of the investment is in the centralization of the fuel and fan controls necessary for semiautomatic operation, it was considered well worth the slight additional cost to superimpose full-automatic control upon the existing semiautomatic system. The balance of the scheme shown in Fig. 7 was recently completed on two boilers and is now under observation in actual operation. Present indications are that it will meet the most rigid requirements as to performance and reliability, that it will respond to emergency load pickups faster than could be accomplished manually, and will also operate satisfactorily on routine load changes as well as maintain perfect stability during extended periods of steady load operation.

Since this system possesses a certain measure of originality in design and performance, which were called for by the exacting nature of the operating requirements, a brief description of its principal features will be presented.

It consists essentially of a ratio controller (embracing the selsyn-operated elements *C*, *D*, and *E* shown in Fig. 7) which sends impulses to the fans and fuel on the basis of a predetermined ratio of turbine steam flow to fuel flow. During sudden load changes, a fuel flow in excess of the steady-load requirements is needed in order to return the heat liberated from the boiler water by means of a reduction in steam pressure and to raise the heat level to the higher point corresponding to the higher load. This overfueling component of the load increment is added through a third link by means of the receiving selsyn *D* which is positioned directly from steam pressure. As the steam pressure returns to normal, this component is reduced to zero and the correct steady-load ratio is again established.

Since the control valves for both gas and oil fuel are calibrated to give a straight-line relation between fuel-valve position and fuel flow, these valves are used as the simplest means of obtaining a positioning impulse proportional to fuel flow through the sending selsyns *S*-1 and *S*-2 shown in Fig. 7. If the load change is fast enough, the fuel valve will respond with a continuous impulse until the new ratio between turbine steam flow and fuel flow has been reached. Then if the steam pressure continues to drop, additional small increments in fuel flow will occur in proportion to the drop in steam pressure.

Correct steam pressure is maintained by means of a differential-type master steam-pressure controller which loads the standard boiler on the basis of steam pressure only. This type of controller is limited with respect to the number of impulses which can be delivered in a given length of time, the maximum being approximately 50 per cent impulses and 50 per cent interruptions. Thus the ratio controller brings the reheat boilers to their final load position twice as fast as the standard boiler is brought to its final position. It was necessary to speed up the drives of the fuel and fan controls to two-and-one-half times their speed as originally installed. This change combined with the substitution of the high-speed ratio control on the reheat boilers made an effective increase of five times the speed of the control as originally installed on these two boilers.

Due to the high speed and the heavy overfueling, it was necessary to add fuel compensation to the steam-flow-air-flow readjusting contactors as shown in Fig. 7. This has the effect of preventing the readjusters from reducing the air flow to correspond with the steam flow until all of the overfueling has been removed, thus preventing a deficiency of air. The standard type of feedwater temperature compensation is also incorporated into the air-flow mechanism, although it is not shown in Fig. 7.

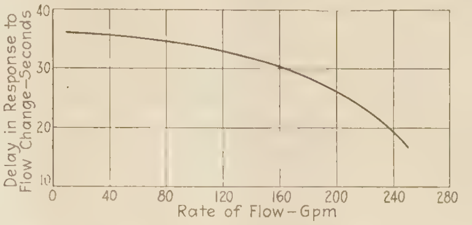


Fig. 9 Delay in Response to Change in Steam Flow to Fuel-Oil Heaters at Various Rates of Oil Flow
(Curves apply to the 1400-lb pressure installation.)

The exacting requirements of the stand-by plant also bring out inherent limitations of other control equipment, such as the control of boiler-water level, fuel-oil pressure and temperature. However, it is entirely possible to find standard equipment that will perform satisfactorily under these conditions. The control of fuel-oil temperature for stand-by conditions introduces a problem of delay in response that is not encountered in a base-load plant. The curve in Fig. 9 shows the variation in this delay from minimum to full load. It is caused partly by the variation in the quantity of heat absorbed by the heater tubes and shell and partly by the variation in oil velocity through the heater over the wide range of loads, resulting in a similar variation in the time required for the body of oil at the changed temperature to reach the temperature bulb of the control. Obviously the control cannot respond until the change in temperature is communicated to the bulb. In order to meet this condition satisfactorily, it is necessary to select a temperature control which is actuated by the rate of temperature change rather than by the amount of difference between neutral and actual temperature. Since there are several standard makes of control on the market which approximate these requirements, a detailed description of any specific type of temperature control would not be justified in this discussion.

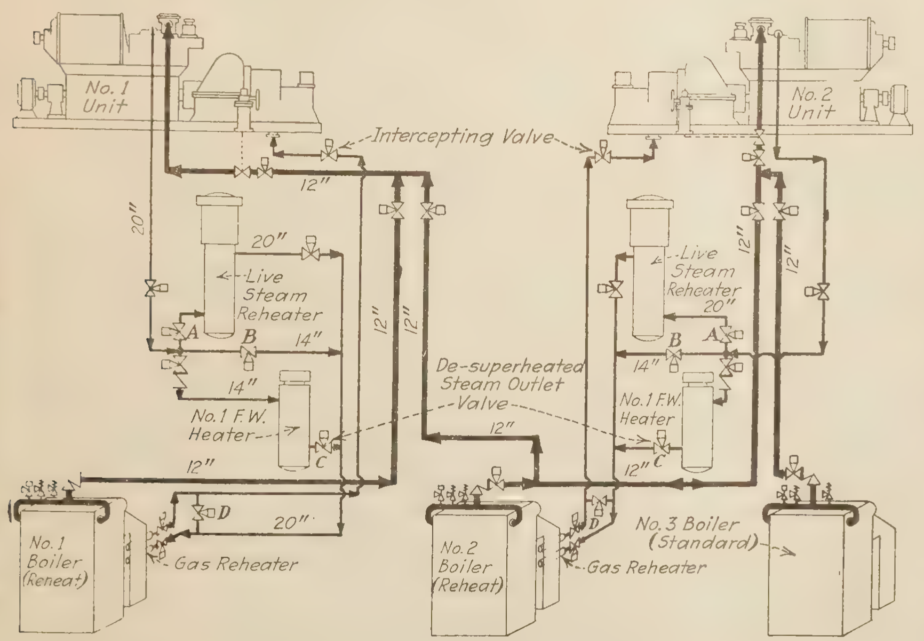


Fig. 8 Arrangement of Boilers and Turbines Used With the 1400-Lb Pressure Installation

### Stand-By Plant-Operating Routine

In order to place stand-by operation upon a routine basis, certain procedures must be established to guarantee reliable performance at all times.

In the first place, an emergency load-pickup schedule must be established on the basis of actual tests. These serve as a guide for the station operators and also for the load dispatcher, and include information as to how much emergency load can be picked up instantaneously from various initial loads, both for the normal setup of boilers and turbines and also for the various conditions where boiler or turbine capacities are curtailed on account of outages of major equipment for annual overhauling or other causes. In Fig. 10 are given typical emergency load-pickup schedules for both oil and gas firing for various operating conditions at a 1400-lb plant. Similar schedules for gas firing are given in Fig. 11 for a 425-lb plant. Since very little fuel oil has been burned in the latter station (see Table 2), no load-pickup tests have yet been conducted with fuel oil.

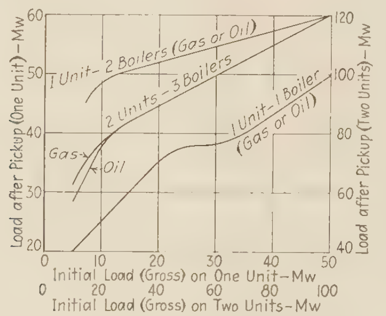


Fig. 10 Emergency Load-Pickup Schedules for the 122,000-Kw, 1400-Lb Plant

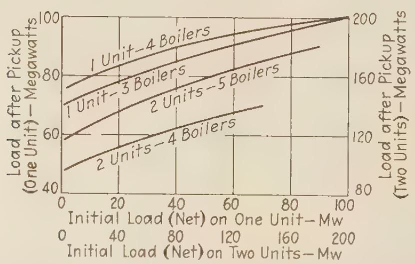


Fig. 11 Emergency Load-Pickup Schedules for the 200,000-Kw, 425-Lb Plant

In the 1400-lb plant, where the load-pickup schedules given in Fig. 10 apply, no load-limiting device is used to restrict the turbine governor to the loads given in the schedules. The governor is left unrestricted to pick up as much load as system speed conditions impose upon the turbine up to the limit of its full capacity at existing steam pressures. The schedules merely indicate the guaranteed minimum that the turbines will handle under the emergency, and include a margin of safety sufficient to allow for unforeseen limitations.

There is another reason for leaving the governor unrestricted by a load-limiting device. It has been found that, by permitting it to open wide whenever speed conditions impose such a demand, the final result is almost always more favorable to the steam plant than if the control-valve stroke were restricted. This is because the initial load can be carried with the steam liberated from boiler-water storage and the result is that, by permitting the turbine to swing to its full capacity initially, there will be a greater recovery in system frequency than would otherwise have been possible, and the net result is usually a lower final load on the turbine after the first few moments. If the boilers have sufficient water storage, such a load may be accepted and held for a few moments even though there is insufficient boiler capacity to sustain it indefinitely.

Table 3 Schedule for Routine Load Reductions on Turbogenerators at Plant *A*

*General.* All load-pickup schedules are based on having the full number of burners in use at all times; this also permits normal operation of the combustion-control system. In order to maintain these conditions while the load on the units is being reduced, it is necessary to allow time for the stored heat in the boiler and turbine equipment to be dissipated before dropping to minimum load. The amount of heat thus stored varies with the load. The following schedule should be considered as the maximum rate of dropping load except when unusual conditions require a faster rate.

*Schedule From Full Load:* (1) Drop to 25,000 kw in steps of 5000 kw every 2 min. (2) Wait 10 min, then drop to 15,000 kw in steps of 5000 kw every 2 min. (3) Wait 10 min, then drop to 10,000 kw. (4) Wait 10 min, then drop to 5000 kw. Total time required to drop from full load to 5000 kw on this schedule is 46 min.

*Schedule From Partial Load.* When dropping from some load less than full load, follow that part of the schedule which applies.

In the process of reducing loads to a minimum, another condition is encountered which can be met by means of a schedule for routine load reductions. Due to the large amount of heat stored in the boiler and turbine equipment, it is necessary, during the process of lowering the load on the boiler, to reduce the fuel rate considerably below the requirements for steady-load conditions. As minimum load is approached, it becomes necessary to cut out burners until steam-pressure conditions permit the fuel rate to be brought up to the normal steady-load rate. For reasons previously explained, cutting out burners reduces the amount of emergency load that may be accepted on account of the delay due to lighting up burners. Thus, if an emergency load demand were to occur during the time some of the burners

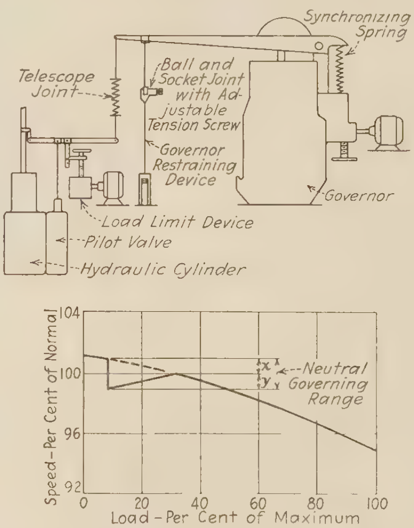


Fig. 12 Throttle-Governor Characteristics With Governor-Restraining Device Installed as Shown

were cut out of service, difficulty would be encountered in picking up the amount of load specified in the emergency load-pickup schedule. It also disrupts the routine of operation. Therefore, the schedule for routine load reductions is established so that the load is not reduced any faster than can be accomplished with all burners in service. Such a schedule is given in Table 3 for the 1400-lb plant. It should be understood that this does not prevent the load from being reduced as fast as desired if operating conditions require it. However, the schedule establishes a routine which may be easily followed and which contributes a surprising amount to the ease of handling load reductions.

One other important device is used in several stations on the Pacific coast, and is known by various names. It will be here

designated as a governor-restraining device. With the large number of hydroelectric plants that form the greater part of most of the generating systems on the Pacific coast, the system speed regulation can be handled most economically at one or more of the hydro plants. Therefore, the governor-restraining device is in common use in the steam plants for establishing a neutral frequency band within which the turbine load is held constant. Such a device is shown in Fig. 12 together with a characteristic curve of governor regulation modified to show the effect of the device when adjusted for a neutral range of plus or minus 1 per cent of normal speed. As soon as the speed falls one per cent below normal, the ball-and-socket joint is separated by the additional pull on the governor beam and the turbine is free to pick up whatever load the speed conditions demand. If the speed exceeds 1 per cent above normal, then the tension on the governor-restraining device is reduced to zero, and a further increase in speed causes the governor to reduce the load on the machine. With the aid of this device, absolutely steady load conditions are maintained for much longer periods of time, with resultant benefits in economy and improved operation without any detrimental effect upon system operating conditions as a whole.

### Load-Pickup Tests in a 1400-Lb Plant

Actual load-pickup tests with gas fuel were first conducted at the 1400-lb plant on one of the 61,000-kw compound units, the general arrangement of which has already been presented in Fig. 8. Details of the equipment are given in Appendix 1 and a sectional elevation of one of the boilers is shown in Fig. 3. At the time of the test, both turbogenerators were in opera-

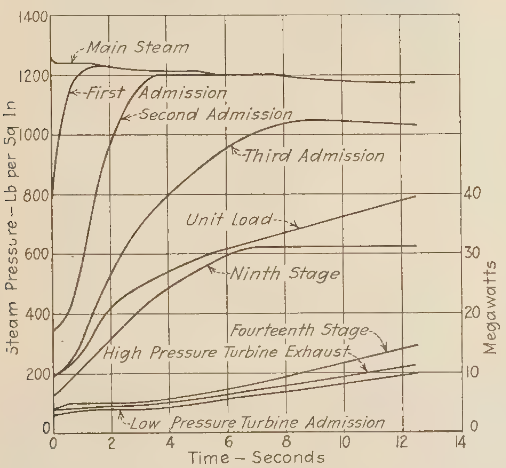


Fig. 13 Throttle and Stage Pressures, and Turbine Loads During the First 12 Sec of Load-Pickup Test on the 1400-Lb Plant

tion and receiving steam from the one standard and two reheat boilers. However, the load pickup was handled by one reheat and one standard boiler. The load on the other reheat boiler and turbine were held constant, although the benefit of the steam-storage capacity from all three boilers was obtained. This showed up very clearly on the boiler flow-meter chart for the reheat boiler which was being held at constant load, a momentary increment in flow from 75,000 to 235,000 lb per hr being recorded although there was no change in the rate of fuel input. However, this dropped back to normal very quickly.

In order to maintain constant frequency during the test, arrangements were made in advance with the load dispatcher to have one or two of the hydroelectric plants stand in readiness to dump sufficient load to maintain the frequency as close to 60 cycles as possible. The restraining device, a sketch of which has already been presented in Fig. 12, was used to block the turbine governor at a steady load of 10,000 kw. The governor-synchronizing spring was then run out to the full-load position, and at a prearranged signal the ball-and-socket connection at *A* was broken, permitting the governor to open the turbine control valves to their wide-open position in 3 or 4 sec. A motion-picture record was made of throttle steam pressure, stage pressures and generator load. These are given in Fig. 13 for the first 12 sec of the test. In Fig. 16 are shown the drum and throttle pressures, fuel-input rates and generator loads for the period of the test. These will be discussed at greater length in an analysis to be presented later.

Superheat and reheat temperatures increased from 725 F to

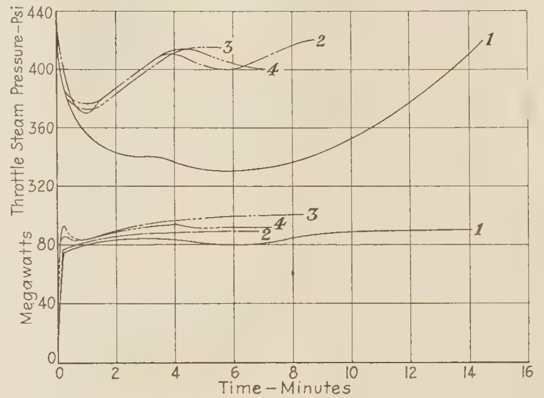


Fig. 14 Throttle Steam Pressures and Turbine Loads During Load-Pickup Tests on the 425-Lb Plant

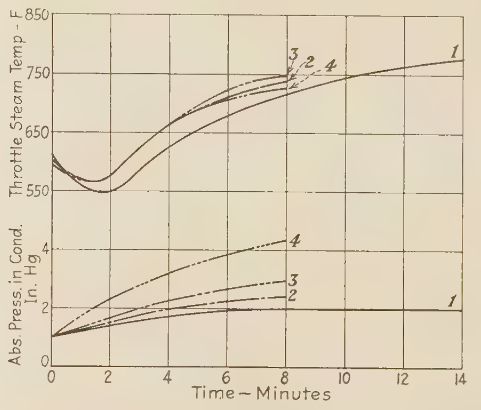


Fig. 15 Throttle Steam Temperatures and Condenser Back Pressures During Load-Pickup Tests on the 425-Lb Plant

approximately 775 F for a few minutes and then returned to about 750 F. It was found that the increase in drum water level was only slightly more than that which is characteristic of the setting of the three-element water-level control, which is adjusted to give approximately $3^1/_2$ in. rise in level from minimum to maximum boiler load. The time required to increase the rate of fuel and air flow from minimum to maximum was approximately 24 sec.

During the first few seconds, the high-pressure generator was slightly overloaded and the differential pressure between the third admission and the ninth stage exceeded the maximum allowable value by approximately $2^1/_2$ per cent. Both of these conditions were of momentary duration and were not

detected by observers during the test, although visible in the motion pictures. There was no evidence of instability of the high-pressure generator in relation to the low-pressure generator.

The stage pressure curves in Fig. 13 show a definite time interval for the steam pressures to build up to their respective maximums in the various stages. As will be shown later, an appreciable amount of energy is absorbed in building up the turbine and extraction-heater system to the new pressure and temperature level corresponding to the increased load.

### Load-Pickup Tests in a 425-Lb Plant

Load-pickup tests with gas fuel were also conducted on one of the 110,000-kw units at the 425-lb plant. Details of the equipment are given in Appendix 2. A sectional elevation of one of the boilers is shown in Fig. 3. Some interesting studies were made of the results that could be obtained under several different operating setups, and are presented herewith.

All tests were made from an initial load of approximately 1500 kw gross. Load was applied by instantaneously raising the pilot-valve lever on the turbine-governing mechanism. An indicating wattmeter was temporarily located near the governor end of the unit so that load could be held constant by manual manipulation of the pilot-valve lever if it were desired to do so.

High-speed clocks were fitted to some of the recording meters for obtaining records while other data were read at timed intervals by observers. High-speed records were made of steam- and feedwater-header pressure in the boiler room, temperature and pressure of steam to turbine throttle, and gas flow.

Although the operators knew at what time the tests were to be made, no changes in the normal stand-by setup were made except for the use of the high-speed operation of the control drives. Also, because of repairs to the fifth-stage heater, it was out of service during all tests and, therefore, no steam was being extracted at that point.

The results of the four principal tests are given in Figs. 14 and 15. Curve 1 shows the performance with three boilers as stand-by capacity. With this arrangement 80,000 kw can be carried on the main generator and station auxiliaries supplied by the house set. However, as the steam pressure drops so low and complete recovery is delayed for 15 min it can be concluded that this setup leaves no margin for safety. If there should be any delay in getting fuel to the boilers or if a fan should fail to start, the performance under emergency stand-by would be greatly jeopardized. To supply reliable stand-by service, more boiler capacity than that provided by three boilers is needed. Use of this setup also allows the steam temperature to reach a very high level before steam pressure is fully restored.

Curve 2 shows the same test conditions as curve 1 but with four boilers sharing the load instead of three. The resultant performance is markedly improved. Time for complete recovery is diminished from 15 min to 4 min, and at the same time a greater load is carried. The drop in pressure shown on the curve after 4 min was due to the firemen cutting back on the fires prematurely.

Curves 1 and 2 were obtained with the main-generator load limited to 90,000 kw and with the house set supplying the station auxiliaries. Curve 3 shows the results of a test made with 4 boilers supplying steam and the turbine admission valves wide open throughout the test with the house set off. Higher load is carried throughout the test period because of the more efficient use of the steam available during the first few minutes. This is due to the generation of the auxiliary power requirements with the main turbine instead of with the house set.

Calculations as shown in Fig. 17 and Appendix 3, revealed the fact that considerable heat left the turbine during the first 2 min to heat the feedwater in the heaters and piping. It was reasoned that, if this heating could be delayed for a few minutes, the same heat could be used for generating power and the feedwater could be heated later after more steam was available from the boilers. Therefore, in an attempt to obtain greater load during the initial phase of the pickup period the extraction valves were throttled until the gates were within $^3/_4$ in. of the bottom of the seats.

The results of such a scheme are shown on curve 4, Figs. 14 and 15. It can be seen that more load is picked up during the first half minute but that it is soon lost and is not readily increased afterward. Since such operation is virtually nonextraction, the excess steam is not readily condensed and the result is a reduction in the vacuum with a consequent loss of load.

The excessive rise of back pressure is clearly shown by the curves in Fig. 15. This might be reduced with the use of two circulators instead of the usual one during an emergency pickup, but the starting of the other circulator is not recommended when the station auxiliaries are being supplied by the house set.

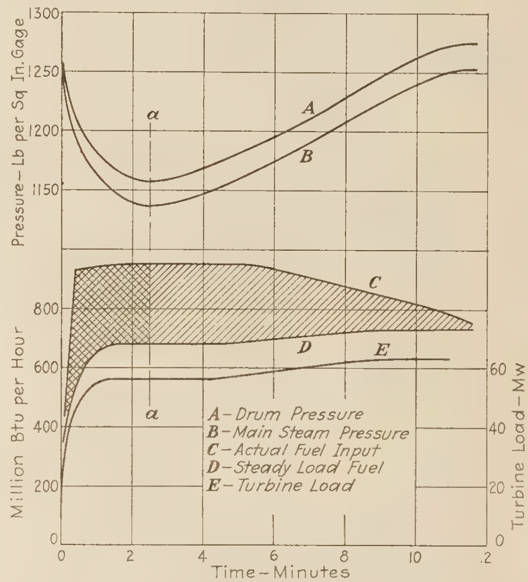


Fig. 16 Drum and Throttle Steam Pressures, Fuel-Input Rates, and Turbine Loads During Load-Pickup Tests From 2 to 90 Megawatts on the 1400-Lb Plant

At the minimum load the boilers are operated with natural draft. Determination of the minimum time required to bring the fans to maximum speed from minimum stand-by position gave an average of 24 sec. This is the time from the closing of the fan switch and pressing the "more" button until the drive has reached the 100 per cent position. Under test conditions, the time required to reach maximum fan speed and maximum fuel flow varied from 34 sec to 1 min after the turbine valves were opened.

When full fuel flow is established in 1 min or less, the effect of opening the feedwater valves, causing an increased flow of water into the boiler and thereby lowering the steam pressure somewhat, is minimized. If the boiler commences to take water at the time of minimum pressure, the restoration of pressure is retarded. For this reason it is well to keep the water level at about 5 in. on the gage when on stand-by so that the addition of water will not be required immediately. On these tests it was found that the water in the boiler would "swell" from an initial level of 5 in. to about 15 in. The swell is only momentary as the increased flow of steam from the boiler soon lowers the water level.

In Fig. 17 are shown the drum and throttle pressures, fuel-input rates and generator loads for the period of the test. These will later be discussed at greater length in connection with the same data already presented in Fig. 16 for the tests on the 1400-lb plant.

The more important conclusions deduced from these tests may be summarized as follows:

(*a*) The use of four boilers instead of three greatly improves the stability of the station and provides a margin of safety so that it is well worth the small additional cost of the fuel necessary to keep the extra boiler on the line.

(*b*) Four boilers can be brought up to maximum rating in practically the same time as three and with no addition to the boiler-room crew.

(*c*) More load can be accepted and held if the extraction valves are not throttled.

(*d*) More load can be accepted and held if the starting of the house set can be delayed without jeopardizing the auxiliary power supply.

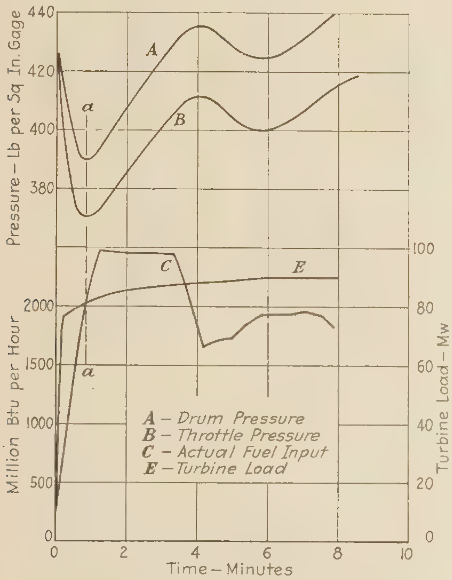


Fig. 17 Drum and Throttle Steam Pressures, Fuel-Input Rates, and Turbine Loads During Load-Pickup Test From 2 to 90 Megawatts on the 425-Lb Pressure Plant

(*e*) If the emergency is such that the house set must be started, 80,000 kw net can be accepted and held; this load can be increased to 85,000 kw after 2 min.

(*f*) A minimum water level of 5 in. on the gage should be maintained at all times when the station is on stand-by.

### Essential Requirements for Stand-By and Base-Load Operation Compared

On the Pacific coast, there are at least two large central stations designed primarily for base-load operation, which have had to operate frequently as stand-by plants. Again, there is the case of a plant designed especially for stand-by service which operated as a base-load plant from the moment it was cut in on the line for a period of approximately one year. Although there were very good reasons for planning the design of these plants for the type of service mentioned, the facts just brought out serve to illustrate the uncertainty involved in any predictions relative to load conditions.

When viewed from the standpoint of control, a plant designed for stand-by operation will also operate more satisfactorily under all other conditions than will a plant which is designed purely for base load. Therefore, it would appear to be desirable to take into consideration in the original design the problem of stand-by operation, even though the immediate future appears to indicate that nothing but base-load operation will be required. In most cases the difference in cost would be relatively insignificant.

The most desirable characteristics for stand-by operation might be summarized as follows:

1 Ease of control over an extremely wide load range. It should be possible to increase the load from minimum to maximum, or vice-versa, either entirely automatically or at least by the manipulation of centralized controls. It is a definite disadvantage if the design is such that burners, pumps, or other equipment must be cut in or out of service during the load change. It is also desirable to design all automatic or semi-automatic control equipment so that it can be relied upon to function properly over the entire load range. The controls should not be so limited in their application that they will only operate satisfactorily within a restricted band of loads. Such limitations, particularly in the case of boiler-water level and combustion control, result in the necessity of depending too much upon human factors for the success of major operations that should proceed smoothly and without even a few seconds delay.

2 High-speed operation of combustion-control equipment for very fast emergency load changes, either fully or semi-automatically, without sacrifice of performance during slow routine load changes or steady load conditions.

3 Adequate manual protection for all automatic functions. The automatic control of combustion, boiler-water level, and fuel-oil temperature and pressure, should be backed up with suitable manual control which can be used with the aid of additional operators to give almost as good results as can be obtained with automatic control. By manual control is meant direct manual manipulation by mechanical means.

4 Dual protection wherever possible for major automatic events such as transitions from one fan-motor speed to another. Thus, it should not be possible for the failure of a single controlling element to cause a failure of major equipment. However, such transitions should be avoided in the design if possible, because the delay during the transition period may restrict the amount of load that can be picked up in an emergency. It also reduces the reliability factor slightly.

5 Superheat and reheat characteristics as flat as possible over a wide load range. This is desirable in order to avoid trouble with flanged joints due to sudden temperature changes in steam piping during emergency load changes.

6 Adequate boiler-water storage should be provided for handling the initial load increment during an emergency. Speed of combustion control should be considered in relation to steam storage available.

7 Both the maximum heat liberation permissible in the furnace and the maximum permissible steam liberation from the boiler without carry-over or other difficulties should be from 35 to 45 per cent greater than steady full-load requirements. This is necessary in order that it may be possible to liberate the additional heat necessary to build the system up to the new temperature and pressure level corresponding to the increased load, and also in order to be able to liberate enough steam from boiler-water storage without carry-over to handle the initial inrush to the turbine.

Items 6 and 7 will be discussed at greater length in the paragraphs to follow.

### Thermodynamic Changes During a Sudden Load Increment

While it is true that the test data represented by the curves of Figs. 16 and 17 together with the values computed therefrom leave much to be desired from the standpoint of exact values, it is felt that the analysis now to be presented will be worth while because it portrays just what takes place during a sudden load increment.

A consideration of the test methods by which the data have been obtained will of course make it necessary to accept the values as approximate only. This, however, will not affect the general picture, and it is questionable whether more precise tests

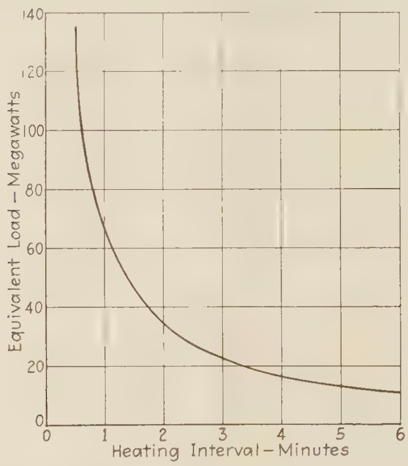


Fig. 18 Load Equivalent of Heat Supplied to Feedwater Heaters During a Sudden Load Increment From 2 to 90 Megawatts

(Curve applies to the 425-lb plant and was plotted from data in Appendix 3.)

would bring to light anything of further value from an operating standpoint. As a research problem, however, it would be of great interest to obtain more precise figures on a number of different installations, and possibly this discussion may suggest such a field for further investigation.

The important phenomena peculiar to this type of operation occur during the period commencing with the opening of the turbine control valves and ending at the moment when the steam pressure stops dropping. In the test represented by Fig. 16 for the 1400-lb plant, this period was $2^1/_2$ min, and in Fig. 17 for the 425-lb plant it was 51 sec. The steam pressure in the boiler drum is shown by curve *A* and the actual fuel-input rates are plotted as curve *C*. The corresponding fuel-input rates for steady load conditions are given by curve *D*. The area between the curves *C* and *D* represents the amount of heat liberation from fuel necessary to increase the heat stored in the boiler setting, turbine, piping, and stage heaters to the new level corresponding to the increased load. Since steam pressure during steady load conditions is held constant at the turbine throttle, the boiler-drum pressure increases with the boiler load and therefore some heat must also be added to the boiler water at the higher loads. There will also be a certain amount for additional losses due to lowered efficiency during the load pickup.

Theoretically, the total amount of heat added to storage in the boiler setting and turbine system is approximately a fixed value for a given load increment and is a definite characteristic of the type and capacity of the installation rather than a function of the time consumed in adding it. However, the time required to add that portion of the heat which is necessary to arrest the drop in pressure is, from an operating standpoint, an important factor which must be considered in relation to the maximum permissible drop in steam pressure. It is also important in connection with the maximum speed for which the combustion control has been designed to operate.

The double-crosshatched area to the left of line *a-a* in Fig. 16 represents the total heat input from fuel that must be added in excess of steady load requirements in order to arrest the drop in steam pressure. In the case of the 1400-lb plant this amounted to approximately 11,000,000 Btu. It should be understood that a portion of this heat is absorbed directly by the boiler setting and the remainder is absorbed for the generation of the steam that is utilized in elevating the temperature and pressure in the turbine, piping, and extraction heaters. The heat liberated from boiler-water storage during this period was 1,950,000 Btu, making a total of 12,950,000 Btu delivered from the two sources, namely, fuel input and boiler-water storage, to storage in the boiler setting and turbine system for the load increment obtained during the test.

From special tests it was found that all of the heat from the incremental fuel input for the first 16 sec was absorbed by the boiler setting. This was obtained by increasing the fuel input by means of a continuous impulse for "more" on the high-speed combustion control while the turbine throttle flow was held constant. The first change in steam pressure or flow of boiler steam was noted at the end of 16 sec, and the control was then reversed and returned to normal. The total fuel input during this period amounted to approximately 1,000,000 Btu. Under the conditions of an actual load pickup, this figure would probably be somewhat higher due to the extra heat that would be absorbed by the various boiler heating surfaces as a result of the increase in flow. However, the error due to the possible lag in the pressure gage and steam-flow meter would probably partially compensate for the error due to disregarding the heat that would be absorbed on account of the flow increments just mentioned. A similar test at the 425-lb plant gave a value of 22 sec.

Thus, the boiler-water storage is the only source available during this period to supply the steam flow to the turbine throttle. At the end of this period, some of the excess fuel represented by the shaded area of Fig. 16 becomes available for reducing the draw-off from boiler-water storage, until finally, at the point of time represented by the line *a-a*, the latter is reduced to zero and the net input to the boiler from fuel (after deducting boiler losses) plus the heat in the feedwater is then exactly equal to the steam output. From this point begins the process of returning the heat to the boiler-water storage plus the heat required to elevate the pressure and temperature of the turbine system from the existing steam pressure to the normal pressure for steady load conditions. Due to the fact that the drum pressure is approximately 21 lb per sq in. higher at the new load, approximately 350,000 Btu additional must be returned to boiler-water storage than was released therefrom during the load pickup.

Due to the unsteadiness of steam-pressure conditions after the load pickup on the 425-lb plant it was unfortunately not possible to plot in Fig. 17 closed areas similar to those in Fig. 16, using the curve of steady-load fuel-input rates to complete the closure. There is, however, one item brought out in the tests at this plant, which should be noted in passing and which was not available in connection with the test on the 1400-lb equipment. In Appendix 3 the amount of heat added to the extraction feedwater heaters during the load pickup has been calculated. This is given for the individual heaters in item 24 and totals approximately 14,500,000 Btu. This is a value of relatively large magnitude and gives some conception as to the amount of heat that has to be added to the turbine system in order to raise its potential to the value corresponding to the new

load. A better appreciation of the magnitude of this value may be had from an examination of the curve in Fig. 18 which shows the average load equivalent of this amount of heat for various time intervals during which the addition of heat might have taken place. If it is accomplished during a 2-min interval, the heat used will correspond to a load of 34,000 kw although the actual equivalent energy delivered is only 1122 kwhr.

The same sort of an analysis is of passing interest in connection with energy and power equivalents of the heat released from boiler-water storage. This is brought out by the curves of Figs. 19 and 20 for a 1400-lb and a 425-lb plant respectively. The values of the electrical-energy equivalents for the amount of heat released by the drop in steam pressure are quite small, but when it is understood that the rates of pressure drop during the course of the load pickup are considerably in excess of 20 per cent per minute, it can be seen that the energy released will sustain an appreciable load for a short interval of time, and this is vital to the success of the sudden load pickup.

### Relations Between Boiler-Water Storage, Fuel-Input Rate and Fuel-Control Drive Speed

A comparison of the steam-pressure curves in Figs. 16 and 17 indicates a much quicker recovery to the point of minimum

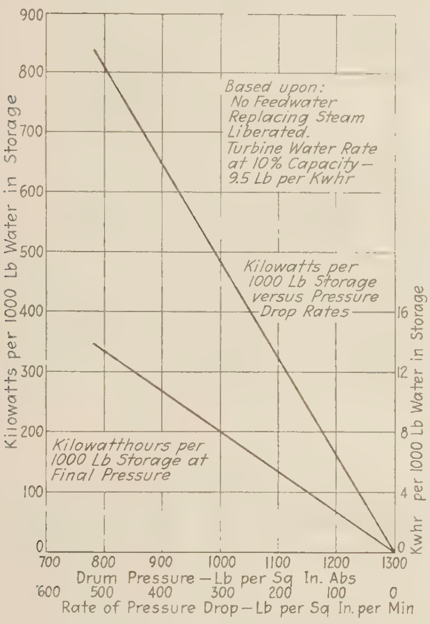


Fig. 19 Kilowatt Loads per 1000 Lb of Boiler-Water Storage for Various Rates of Pressure Drop, and Kilowatthours for Various Total Pressure Drops, for the 1400-Lb Pressure Installation

steam pressure in the case of the 425-lb plant. This was due almost entirely to the difference in the relative amounts of excess fuel-input rates above steady load requirements. In the case of the 1400-lb plant, no effort was made to utilize the maximum available fuel-input rate. For the conditions of the test, the amount of this excess could easily have been doubled. However, from calculations made previous to the test, the maximum fuel-input rates indicated in Fig. 16 were decided upon as being ample to effect a complete recovery of steam pressure within a reasonable length of time. Therefore, the operators were instructed to run the fuel-control drive to a predetermined position and hold it there until minimum steam pressure was reached.

A study of the curves in Fig. 16 will justify within reasonable limits certain general conclusions as to the relation between excess fuel-input rates, fuel-control drive speeds and boiler-water storage. A decrease in the excess fuel-input rate above steady-load requirements, or the amount of overfueling, will delay the moment when the total heat input from fuel in excess of steady-load requirements has reached the value represented

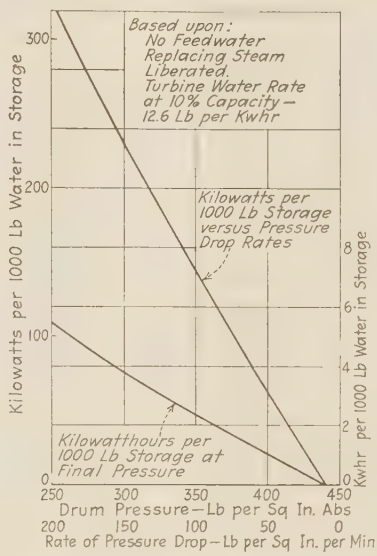


Fig. 20 Kilowatt Loads per 1000 Lb of Boiler-Water Storage for Various Rates of Pressure Drop, and Kilowatthours for Various Total Pressure Drops, for the 425-Lb Pressure Installation

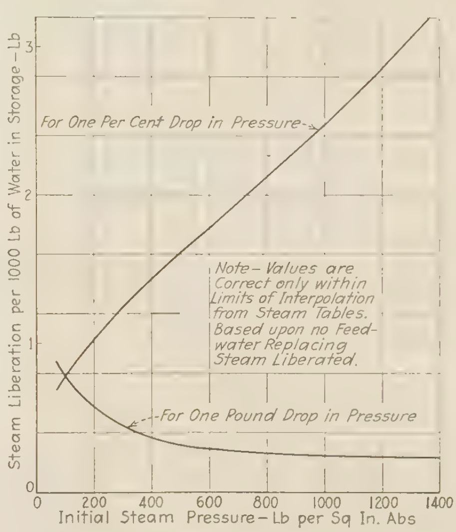


Fig. 21 Comparison of Steam Liberated From Boiler Water Storage for 1 Lb and 1 Per Cent Pressure Drops at Various Initial Pressures

by the double-crosshatched area. This will make the moment when minimum steam pressure has been reached occur later and more steam will have been drawn from boiler-water storage to meet the throttle flow requirements, thus resulting in a lower minimum steam pressure. If, for example, the amount of overfueling were reduced 20 per cent, then 20 per cent of 11,000,000 Btu or an additional 2,200,000 Btu would have to be withdrawn

TABLE 4 COMPARISON OF MAXIMUM TURBINE CAPACITIES AVAILABLE MOMENTARILY FROM STORED HEAT IN BOILER WATER FOR FOUR PACIFIC COAST STEAM STATIONS

| Plant | Initial boiler-drum pressure, lb per sq in. gage | | Maximum capacity, kw | Turbine water rate at 10% max capacity, lb per kwhr | Total boiler storage at normal operating temperature, lb | Capacity from storage for 20% pressure drop per min, kw | Proportion of max turbine capacity from storage, per cent |
|---|---|---|---|---|---|---|---|
| *A* | 1285 | (*a*) | 122,000 | 9.5 | 126,000 | 48,000 | 39.5 |
| | | (*b*) | 61,000 | 9.5 | 126,000 | 48,000 | 79.0 |
| *B* | 425 | (*a*) | 220,000 | 12.6 | 990,000 | 142,000 | 65.0 |
| | | (*b*) | 110,000 | 12.6 | 660,000 | 95,000 | 86.0 |
| *C* | 450 | | 42,000 | 13.0 | 306,000 | 42,500 | 104.0 |
| *D* | 425 | | 44,000 | 13.5 | 256,000 | 34,000 | 80.0 |

NOTE: Item (*b*) for plant *A* is based upon a load pickup with one turbine and two boilers but with the benefit of storage capacity from three boilers. This is the necessary condition for a pickup from 5 megawatts to 56 megawatts.
NOTE: Item (*b*) for plant *B* is based upon a load pickup with one turbine and four boilers, which is the condition necessary for a pickup from 2 megawatts to 90 megawatts.

TABLE 5 COMPARISON OF RATIOS BETWEEN INSTALLED BOILER AND TURBINE CAPACITIES FOR FOUR PACIFIC COAST STEAM STATIONS

| Plant | Operating pressure at turbine throttle, lb per sq in. | Operating temperature at turbine throttle, F | | Maximum installed turbine capacity, kw | Turbine water rate at max capacity, lb per kwhr | Maximum installed boiler capacity 1000 lb per hr | Maximum installed boiler capacity Equiv. kw | Ratio boiler to turbine capacity, col 7/col 4 |
|---|---|---|---|---|---|---|---|---|
| *A* | 1250 | 750[a] | (*a*) | 122,000 | 8.9 | 1,500 | 169,000 | 1.39 |
| | | | (*b*) | 61,000 | 8.9 | 970 | 109,000 | 1.79 |
| *B* | 400 | 725 | (*a*) | 220,000 | 10.5 | 2,700 | 257,000 | 1.17 |
| | | | (*b*) | 110,000 | 10.5 | 1,800 | 171,000 | 1.56 |
| *C* | 425 | 725 | | 42,000 | 11.5 | 850 | 74,000 | 1.76 |
| *D* | 400 | 650 | | 44,000 | 10.8 | 850 | 79,000 | 1.79 |

[a] With reheat to 750 F.
NOTE: Item (*b*) for plant *A* is based upon two boilers taking the load increment on one turbine, with the other turbine operating at minimum load. This was the actual condition for a load pickup from 5 megawatts to 56 megawatts.
NOTE: Item (*b*) for plant *B* is based upon one turbine operating with four boilers, which is the arrangement necessary for a pickup from 2 megawatts to 90 megawatts.

from boiler-water storage. The new minimum steam pressure can therefore be computed, also the approximate time of its occurrence.

On the other hand, if the fuel-control drive speed were slowed down, this would change the slope of the fuel-input line toward the right and delay the start of the overfueling process, thus necessitating a larger draw-off from boiler-water storage, which in turn results in a lower steam pressure. However, the time of occurrence of minimum steam pressure will not be altered a great deal due to the fact that the excess fuel-input rate to the boiler setting will be increased by approximately the amount of the reduction in heat available for steam generation, thus compensating largely for the delay in reaching the desired maximum fuel rate.

If sufficient data could be obtained as to the value of the total heat per kilowatt of full-load capacity in excess of steady-load requirements (or some other more suitable unit basis) which is required to arrest the pressure drop, then it would be a simple matter to design for the most suitable relations between boiler-water storage, maximum possible fuel-input rate, and maximum fuel-control drive speed. There are five factors involved as follows:

(*a*) Total heat in excess of steady-load fuel requirements necessary to arrest the steam-pressure drop for a given load increment. It is dependent almost entirely upon the size and type of equipment installed. However, much more actual test data are required than are now available.

(*b*) Maximum permissible steam-pressure drop considered safe or desirable. This will usually be between 15 and 20 per cent of the initial pressure.

(*c*) Maximum permissible fuel-input rate. This is largely a matter of permissible first cost as dictated by the economics of the situation.

(*d*) Maximum possible speed of fuel-control-drive equipment. This will depend largely upon the type of combustion-control equipment selected.

(*e*) Total boiler-water storage at normal firing level and operating pressure.

Since item (*a*) is a fixed quantity dependent upon the size and type of equipment installed, and since any three of the remaining four items can be fixed arbitrarily on the basis of a compromise between available equipment and first cost, the remaining factor could be calculated to meet the other fixed requirements.

## Boiler Capacities and Storage Factors for Four Stations on the Pacific Coast

Since the available data in connection with item (*a*) as just presented are insufficient for practical use at this time, some design characteristics for four stations on the Pacific coast will now be analyzed in the light of the results obtained from actual load-pickup tests.

In Table 4, the kilowatts equivalent to the heat liberation from the boiler-water storage for a pressure-drop rate of 20 per cent per minute have been computed for the four steam stations. In the last column these figures have been compared with the full-load capacities of the turbines on a percentage basis. In item (*b*) of this column for plants *A* and *B*, are given the values of these boiler-water storage factors, which correspond to the actual conditions of the load-pickup tests described earlier in this discussion. It will be recalled that plant *A* in the test received the benefit of the storage capacity from all three boilers when picking up the load on only one turbine. The proportion of full-load capacity that could be carried from storage under these conditions is 79 per cent. In the case of plant *B*, the tests showed that three boilers per turbine did not give very satisfactory results inasmuch as the drop in steam pressure was excessive. However, with four boilers per turbine, very good results were obtained, and the boiler-water storage factor for this arrangement is 86 per cent of the full-load capacity of the turbine. This is somewhere near the value of 79 per cent for plant *A*.

In item (*a*) for each of these two plants, the boiler-water storage factor is given on the basis of all boilers and turbines in service. For plant *A* the figure of 39.5 per cent is for three boilers and two turbines, and for plant *B* the value of 65 per cent applies either to six boilers and two turbines or three boilers and one turbine. It may be concluded from the results of the load-pickup tests that both of these figures are too low for satisfactory stand-by operation, if by this is meant the capacity to accept and hold approximately full load from an initial minimum load. This condition is also reflected in the emergency load-pickup schedules plotted in Figs. 10 and 11.

It may further be concluded from this analysis that the minimum boiler-water storage factor for satisfactory stand-by operation should be somewhere between 65 and 79 per cent. It is also interesting to note the very large factor indicated for plant *C*. The figure for plant *D* is of the right order. However, contemplated additions in turbine capacity at both these plants will reduce the storage factor to a value that will be too low for a full-load emergency pickup unless additional boiler capacity is also installed.

In passing, it is of interest to note the difference in the values of actual boiler-water storage as given in column 5. If these values are reduced to a unit basis by dividing them by the corresponding kilowatt capacities in column 3, a value of approxi-

mately 2 lb of storage per kilowatt of capacity is obtained for item (*b*) of plant *A* as against 6 lb per kilowatt for item (*b*) of plant *B*, although the corresponding storage factors in column 7 are of the same order. This merely indicates that the same operating results may be obtained from much less boiler-water storage at the higher pressure. Although the lower water rate for the 1250-lb turbine will account for part of this, reference to Fig. 21 reveals the fact that, for the same per cent drop in steam pressure, more than twice as much steam is liberated from the same quantity of water at 1250 lb pressure as is possible at 400 lb.

In Table 5 are given the ratios of boiler capacity in equivalent kilowatts to the actual turbine capacity. All plants exceed the minimum requirements of 35 per cent excess boiler capacity, as called for under item 7 in the list of essential requirements for stand-by operation presented earlier in the discussion. The one exception to this is indicated in item (*b*) for plant *B*, which is for three boilers and one turbine or six boilers and two turbines.

### Acknowledgments

For various reasons, it has been considered preferable to withhold the identity of the various stations and companies referred to herein. However, the author wishes to make it clear that some very valuable portions of the material presented would not have been available without the generosity and cooperation of several engineers in companies other than the one in which he is employed. The assistance that has been rendered in this way is gratefully appreciated.

## Appendix 1

### Boilers and Auxiliaries of the 1400-Lb Plant

The boilers used in the tests are Babcock and Wilcox cross-drum, straight-tube type with superheater located above boiler sections. The furnace is enclosed on four sides by Bailey insulated waterwalls. Two of the boilers are of the reheat type, as shown in Fig. 8; the third boiler is of the standard type. Each boiler is equipped with a return-bend economizer and tubular air heater, one forced-draft fan and one induced-draft fan. Boiler and furnace data are as follows:

| Boiler and furnace: | Reheat boilers | Standard boiler |
|---|---|---|
| Maximum evaporating capacity, lb per hr | 500,000 | 500,000 |
| Steam-generating surface, sq ft | 12,300 | 12,300 |
| Furnace volume, cu ft | 15,100 | 15,100 |
| Tube, outside diameter, in | 4 | 4 |
| Maximum operating pressure, lb per sq in., gage | 1,400 | 1,400 |
| Weight of water at normal operating level and temperature, lb | 42,000 | 42,000 |
| Superheater, convection, single pass: | 4 loops | 5 loops |
| Surface, sq ft | 6,300 | 7,340 |
| Tube, outside diameter in | 2 | 2 |
| Primary reheater (steam to steam) located above boiler so that drips return to boiler drum by gravity: | | |
| Surface, sq ft | 2,700 | None |
| Tube, outside diameter, in | 2 | None |
| Outside diameter of shell, in | 60 | None |
| Length of shell, in | 12.5 | None |
| Secondary reheater, convection, single-pass, located in second pass of boiler: | | |
| Surface, sq ft | 9,560 | None |
| Tube, outside diameter, in | 2 | None |
| Economizer, return-bend loop type: | | |
| Surface, sq ft | 17,900 | 21,100 |
| Tube, outside diameter, in | 2 | 2 |
| Air heater, tubular: | | |
| External heating surface, sq ft | 51,600 | 46,915 |
| Internal heating surface, sq ft | 46,650 | 42,413 |
| Tube, outside diameter, in | 2 | 2 |

Sturtevant fans are used in the 1400-lb plant. They are inlet-vane control-type, direct-connected to two constant-speed induction motors, and are arranged for automatic transition from low- to high-speed motor, and vice versa, and for automatic high-speed readjustment of inlet vanes during fan-motor transition. The data on these fans are as follows:

| Induced-draft fans: | Reheat boilers | Standard boiler |
|---|---|---|
| Capacity, cfm at 385 F | 290,000 | 240,000 |
| Speed, rpm | 876 | 872 |
| High-speed motor, rpm | 900 | 900 |
| High-speed motor, hp | 1,200 | 800 |
| Low-speed motor, rpm | 600 | 600 |
| Low-speed motor, hp | 350 | 225 |
| Forced-draft fans: | | |
| Capacity, cfm at 80 F | 165,000 | 135,000 |
| Speed, rpm | 700 | 870 |
| High-speed motor, rpm | 720 | 900 |
| High-speed motor, hp | 600 | 550 |
| Low-speed motor, rpm | 450 | 600 |
| Low-speed motor, hp | 150 | 150 |

Peabody-Fischer wide-range mechanical oil burners, and three-piece natural-gas burners are used in the 1400-lb plant. Burner data are:

| | Reheat boilers | Standard boiler |
|---|---|---|
| Number per boiler | 26 | 22 |
| Natural-gas capacity per burner at 8 lb per sq in., cu ft per hr | 21,600 | 21,000 |
| Fuel-oil capacity per burner at 275 lb per sq in., lb per hr | 1,525 | 1,525 |

### Turbogenerators and Auxiliaries

The turbogenerators are of the General Electric vertical compound type, consisting of a high-pressure turbogenerator mounted on the stator of the low-pressure turbogenerator. Other data are as follows:

| | |
|---|---|
| High-pressure turbine: | |
| Speed, rpm | 3,600 |
| Number of stages | 14 |
| Normal throttle pressure, lb per sq in., gage | 1,250 |
| Total temperature of steam, F | 750 |
| Exhaust pressure, lb per sq in., gage | 30–425 |
| Generator capacity, kw | 13,000 |
| Low-pressure turbine: | |
| Speed, rpm | 1,800 |
| Number of stages | 17 |
| Normal throttle pressure, lb per sq in., gage | 25–405 |
| Normal temperature of steam, F | 705–750 |
| Exhaust pressure, in. Hg, abs | 1.25 |
| Generator capacity (compound operation), kw | 48,000 |
| Shaft exciter direct-connected to generator, kw | 200 |
| Capacity of compound unit, kw | 61,000 |
| Name-plate capacity, kw | 50,000 |

Steam is extracted from cross-over line between high-pressure and low-pressure turbines to secondary feed-pump turbines and heater No. 1. Exhaust from the former supplies heater No. 2. Extraction from the 10th stage to heater No. 3 and evaporators. Vapor from evaporators and gland leak-off to deaerating heater No. 4. Extraction from 14th stage to heater No. 5.

## Appendix 2

### Boilers and Auxiliaries of the 425-Lb Plant

The boilers used in the tests described are Babcock and Wilcox cross-drum, straight-tube type with an interdeck superheater. The furnace is completely inclosed by Bailey insulated waterwalls. Each boiler is provided with a tubular air heater, two

forced-draft fans and one induced-draft fan. Other data are as follows:

| | |
|---|---|
| Boiler and furnace: | |
| Boiler horsepower | 3,416 |
| Heating surface, sq ft | 34,160 |
| Total furnace volume, cu ft | 19,800 |
| Tube, outside diameter, in | 4 |
| Water at normal operating level, gal | 24,000 |
| Water at normal operating level and temp, lb | 165,000 |
| Superheater: | |
| Make | B & W |
| Type—convection, single-pass, four-loop, quadruple-deck: | |
| Surface, sq ft | 6,588 |
| Tube, outside diameter, in | 2 |
| Air heater: | |
| Make | B & W |
| Type | Tubular |
| External heating surface, sq ft | 51,232 |
| Internal heating surface, sq ft | 46,350 |
| Tube, outside diameter, in | $2^1/_2$ |
| Number of tubes | 2,176 |
| Burners: | |
| Number per boiler | 20 |
| Natural-gas capacity per burner at 5 lb per sq in., cu ft per hr | 29,250 |
| Boiler rating for gas flow, per cent | 425 |

The fans used in the 425-lb plant are driven by variable-speed brush-shifting motors. Peabody-Fischer wide-range mechanical oil-burning type and three-piece natural-gas burners are used.

### Turbogenerators

The turbine and generators were manufactured by the General Electric Company. They are of the two-cylinder tandem-compound type with double flow in the low-pressure cylinder. Steam is extracted from the 5th, 10th, 14th, and 18th stages. Other data are as follows:

| | |
|---|---|
| Turbine: | |
| Speed, rpm | 1,500 |
| Normal throttle pressure, lb per sq in., gage | 400 |
| Total temperature of steam at throttle, F | 750 |
| Exhaust pressure, in. Hg abs | 1.5 |
| Number of stages in high-pressure cylinder | 18 |
| Number of stages in low-pressure cylinder | 3 |

The main generator has rated capacity of 90,000 kw at a power factor of 90 per cent. The auxiliary generator has a rated capacity of 4000 kw at 80 per cent power factor. The actual capacity of the unit is 110,000 kw.

## Appendix 3

Calculation of power equivalent of heat added to feedwater heaters during a sudden load increase from 2 to 90 megawatts, gross. It is assumed that there is no flow through the heaters during the heating period, and that the evaporator is initially on live steam but transient heating is supplied by steam from the tenth stage.

| | Heater | 5th | 10th | EC | 14th | 18th |
|---|---|---|---|---|---|---|
| 1 | Weight of heater filled, lb | 75,000 | 68,000 | 50,000 | 52,000 | 53,800 |
| 2 | Weight of heater empty, lb | 61,000 | 55,000 | 36,800 | 36,000 | 37,000 |
| 3 | Weight of water in heater, lb | 14,000 | 13,000 | 13,200 | 16,000 | 16,800 |
| 4 | Tube surface, sq ft | 3,300 | 2,850 | 3,200 | 4,000 | 4,500 |
| 5 | Number of tubes | 1,996 | 1,996 | 2,220 | 2,368 | 2,536 |
| 6 | Surface per tube, sq ft | 1.65 | 1.43 | 1.44 | 1.69 | 1.77 |
| 7 | Outside diameter of tube, in | 0.75 | 0.75 | 0.625 | 0.625 | 0.625 |
| 8 | Gage of tube, Bwg | 14 | 14 | 18 | 18 | 18 |
| 9 | Effective length of tube, in. Item 6/(3.1416 × item 7/12) | 8.4 | 7.2 | 8.8 | 10.3 | 10.8 |
| 10 | Tube-sheet thickness, ft | 0.33 | 0.33 | 0.12 | 0.17 | 0.17 |
| 11 | Total length of tube, ft. (2 × item 10) + item 9 | 9.1 | 7.9 | 9.0 | 10.6 | 11.1 |
| 12 | Weight of tube, lb per ft | 0.64 | 0.64 | 0.33 | 0.33 | 0.33 |
| 13 | Total weight of tubing, lb. Item 5 × item 11 × item 12 | 11,600 | 10,100 | 6,630 | 8,300 | 9,300 |
| 14 | Weight of shell, lb. Item 2 — item 13 | 49,400 | 44,900 | 30,200 | 27,700 | 27,700 |
| 15 | Temperature of water leaving heater at 2 megawatts load, F | 220 | 175 | 165 | 85 | 80 |
| 16 | Temperature of water entering heater at 2 megawatts load, F | 175 | 165 | 85 | 80 | 70 |
| 17 | Average temperature in heaters, F | 197 | 170 | 125 | 82 | 75 |
| 18 | Temperature of water in heaters at a load of 90 megawatts, F | 388 | 320 | 260 | 249 | 169 |
| 19 | Temperature rise due to load, F. Item 18 — item 17 | 191 | 151 | 135 | 167 | 94 |
| 20 | Heat to brass, Btu. Item 19 × item 13 × 0.092[a] | 204,000 | 139,000 | 82,000 | 127,000 | 80,000 |
| 21 | Heat to steel, Btu. Item 19 × item 14 × 0.118[b] | 1,110,000 | 795,000 | 481,000 | 546,000 | 308,000 |
| 22 | Heat to water, Btu. Item 19 × item 3 | 2,680,000 | 1,950,000 | 1,780,000 | 2,680,000 | 1,580,000 |
| 23 | Total heat absorbed by heater, Btu. Item 20 + item 21 + item 22 | 3,994,000 | 2,884,000 | 2,343,000 | 3,353,000 | 1,968,000 |
| 24 | Heat content of steam at extraction point, Btu per lb | 1,321 | 1,246 | 1,170 | 1,163 | 1,073 |
| 25 | Heat content of drips, Btu per lb | 365 | 291 | 225 | 216 | 135 |
| 26 | Heat content of extracted steam less heat content of drips, Btu per lb. Item 24 — item 25 | 956 | 955 | 945 | 947 | 938 |
| 27 | Steam extracted, lb. Item 23/item 26 | 4,180 | 3,020 | 2,480 | 3,540 | 2,100 |
| 28 | Heat drop, stage to condenser, Btu per lb | 349.5 | 275 | 275 | 191.5 | 101.5 |
| 29 | Kilowatthours lost to heaters. (Item 28 × item 27)/3414 | 418 | 244 | 200 | 199 | 61 |
| 30 | Total kilowatthours lost to all heaters | | | | | 1,122 |

[a] Specific heat of brass.
[b] Specific heat of steel.
Note: Values in this table used in plotting Fig. 18.

# Development of a Fuel-Injection Spark-Ignition Oil Engine

By NICHOLAS FODOR,[1] MILWAUKEE, WIS.

**The author describes a fuel-injection spark-ignition oil engine which was designed to operate on the same fuel as a Diesel engine and with approximately the same fuel economy, but with pressures throughout the entire cycle which would not exceed those of a gasoline engine of equal size and speed designed for the same service.**

**In the oil engine only air is admitted to the cylinder during the suction stroke. This air is then compressed to 160 lb per sq in., corresponding to about 6.35 to 1 compression ratio. At the end of the compression stroke, at about 60 deg before top dead center, the fuel is injected into the cylinder in finely atomized state by the fuel pump and injector. The fuel pump and injector are of the same design as those used for Diesel engines. Injection continues until the piston reaches top dead center. The spark occurs 15 deg before top dead center. Due to the time necessary for the vaporization of the injected fuel and its mixing with the air inside the cylinder, not all the fuel injected during the time from the beginning of injection until the spark occurs, will ignite at once, but a rather orderly combustion will take place and the maximum pressure is reached after 10 to 12 deg top dead center, when the engine is running at its rated speed and carries full load. At part-load operation, the beginning of injection is correspondingly later, but the spark timing is fixed. The rate of combustion is controlled by the rate of discharge of the fuel.**

**The expansion and the exhaust strokes have the same functions in this engine as in gasoline and Diesel engines.**

**As in the case of the gasoline engine, the spark can ignite a mixture in the fuel-injection spark-ignition oil engine only when it is constituted of the proper amount of fuel and air necessary to sustain combustion, i.e., a mixture must have the proper air-fuel ratio. In order to maintain this proper air-fuel ratio at all loads, it is necessary that the quantities of fuel and air be controlled in accordance with the momentary load.**

**At full load the total quantity of air available in the engine is utilized and an air-fuel ratio of approximately 16 to 1 is maintained. At part loads the air admitted to the cylinders is reduced in accordance with the momentary load. For metering the fuel, the vacuum of the manifold is used, which acting upon the fuel pump will set the corresponding quantity of fuel.**

**Electric current for the spark plug is supplied from either a magneto or a battery by the customary coil and distributor.**

**The fuel consumption at full load of the production engine amounts to 0.44 lb per bhp-hr, and the part-load fuel economy approaches very closely that of the Diesel engine. The engine can run on gasoline, on fuel oils used in house-heating furnaces, or on regular fuel oils used for Diesel engines. The engine seems to be very insensitive toward its fuel.**

**The engine has a very flat torque characteristic and is highly flexible, therefore, ideal for automotive use.**

**Inasmuch as the maximum pressures are the same as those in gasoline engines, the fuel-injection spark-ignition oil engine is built with the same weight per horsepower as that of the gasoline engine.**

The purpose of this paper is to explain briefly the principal features of the spark-ignition fuel-injection oil engine developed in the oil-engine laboratory of the Allis-Chalmers Manufacturing Company, and to present the fundamentals of combustion of such an engine.

The development of the oil-engine was prompted by the demand for replacing gasoline as fuel in tractor engines, mainly, on account of the greater economy of operation. The adoption of the Diesel engine in place of the gasoline engine seemed to be plausible when the change was considered, but after exhaustive tests it was found that existing transmissions of the tractors could not stand up for any length of time under Diesel operation. The high cyclic torque variations of the Diesel engine were detrimental to the clutches, gears, and shafts of the transmission, and had the Diesel engine been adopted for the tractor, it would have been necessary to redesign the entire transmission which would have materially increased the cost to the consumer. Due to the fact that the Allis-Chalmers Manufacturing Company is a large producer of gasoline-engined tractors, the splitting of the production line into separate sections for gasoline- and Diesel-engined tractors would be very undesirable from an economical standpoint and, therefore, the company decided to investigate the possibilities of the low-compression, fuel-injection, spark-ignition oil engine.

A comparison of indicator cards of Allis-Chalmers oil engines and Diesel engines of equal bore and stroke is shown in Fig. 1. The oil-engine diagram shows an actual power diagram of the oil engine when running at 1050 rpm with an indicated mep of 111 lb per sq in. The Diesel diagram has been computed on the basis of a compression ratio of 16 to 1, an indicated mep of 111 lb per sq in. and a maximum pressure of 970 lb.

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Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until March 10, 1937, for publication at a later date. Discussion received after the closing date will be returned.

Fig. 2 shows the tangential diagram of a $5^1/_4 \times 6^1/_2$-in. four-cylinder 1050-rpm Allis-Chalmers oil engine and the tangential diagram of a Diesel engine of the same size—running at the same speed. Tangential forces were calculated from the diagrams shown in Fig. 1.

Fig. 3 shows the diagrams of actual torque delivery at the flywheel at 1050 rpm by six-cylinder oil and Diesel engines. These diagrams illustrate the extent of the cyclic variation of torque for both types of engines, and they are self-explanatory.

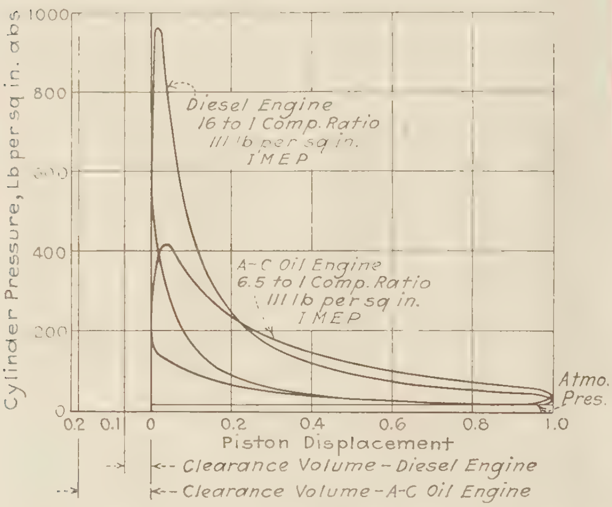


Fig. 1 Comparison of Indicator Cards of the A-C Oil Engine and a Diesel Engine of the Same Size

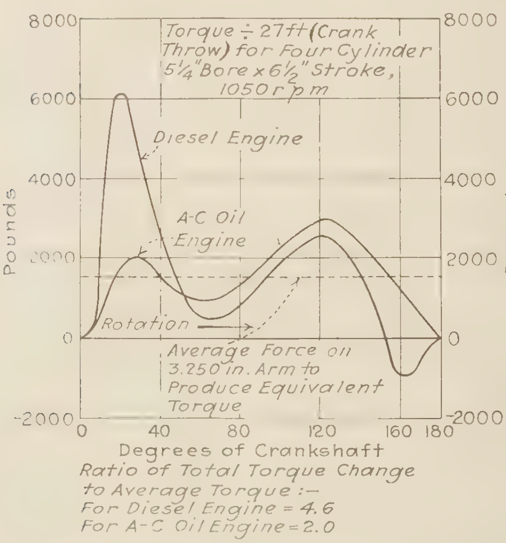


Fig. 2 Tangential Diagrams of the A-C Oil Engine and Diesel Engine of the Same Size

Before starting the development of the fuel-injection spark-ignition oil engine, all pertinent factors were considered and analyzed and as a result, it was found that a desirable oil engine for tractors, and for that matter for all other automotive purposes, should fulfill the following requirements:

1 It must burn the same low-cost fuel oils as used in commercial Diesel engines, and with approximately the same fuel economy.

2 It must not be sensitive to fuels and should operate on any hydrocarbon fuel which will flow easily at surrounding temperatures.

3 Its cylinder pressures throughout the entire cycle of operation must not exceed those of the gasoline engine of identical dimensions, speed and compression ratio, and designed for the same service.

4 It must start immediately when cold without artificial means or complicated starting mechanisms.

5 Construction of the elements of the engine and the material employed should be identical with those of the gasoline engine.

6 The fuel-injection system must be simple, its operation easily understandable to the average mechanic, and its servicing must require neither special tools, equipment, nor training. The same specifications refer to the ignition system.

Fig. 4 shows a view of the oil engine; Fig. 5 is a cross section of the same engine.

## Principles of Operation

The principles of operation of the Allis-Chalmers oil engine are as follows: It operates on the four-cycle principle with air

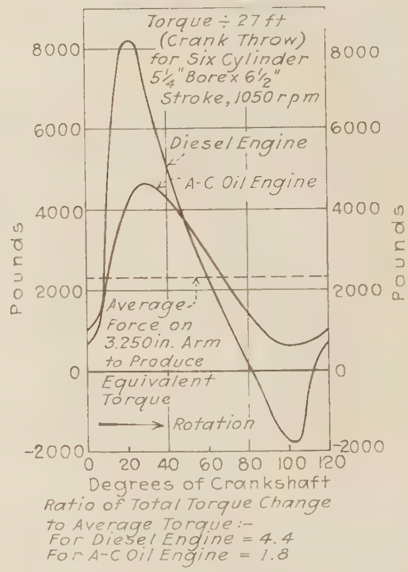


Fig. 3 Torque Delivery at the Flywheel of the A-C Oil Engine and a Diesel Engine of the Same Size, Both Running at 1050 Rpm

only being admitted to the cylinders during the intake stroke. This air is then compressed to a pressure of 160 lb per sq in., corresponding to a compression ratio of approximately 6.35 to 1. Toward the end of the compression stroke, at about 50 deg before top dead center, fuel injection commences and fuel is injected into the cylinders in a finely atomized state by the fuel pump and injector. The fuel pump and injector are of a design similar to those used in Diesel engines. Injection continues until 10 deg before top dead center. The spark occurs about 12 deg before top dead center. Due to the time necessary for the vaporization of the injected fuel and its mixing with air inside the cylinder, not all the fuel injected during the interval from the beginning of the injection until occurrence of the spark, will ignite at once, but a rather orderly combustion takes place and the maximum pressure is reached at approximately 15 to 18 deg past top dead center, when the engine is operating at its rated speed and load. At part-load operation, the beginning of the injection is correspondingly later, but the spark timing is fixed.

The rate of combustion is controlled by the timing of the fuel injection.

The expansion and exhaust strokes have the same functions in the oil engine as in the gasoline or Diesel engines.

As in the case of a gasoline engine, the spark can ignite a mixture in the fuel-injection spark-ignition oil engine only when it consists of the proper mixture of fuel and air to sustain combustion, i.e., the mixture must have the proper air-fuel ratio. In order to maintain this proper air-fuel ratio at all loads, it is necessary that not only the fuel quantity be controlled in accordance with the load, but also the air.

At full load the total quantity of air available in the engine is utilized and an air-fuel ratio of 15 or 16 to 1 is maintained. At all part loads, the air admitted to the cylinders is reduced in accordance with the momentary load. This is accomplished by a simple butterfly valve placed between the air-cleaner and intake manifold and operated by a conventional governor. For metering the fuel, the vacuum of the manifold is used to regulate the fuel

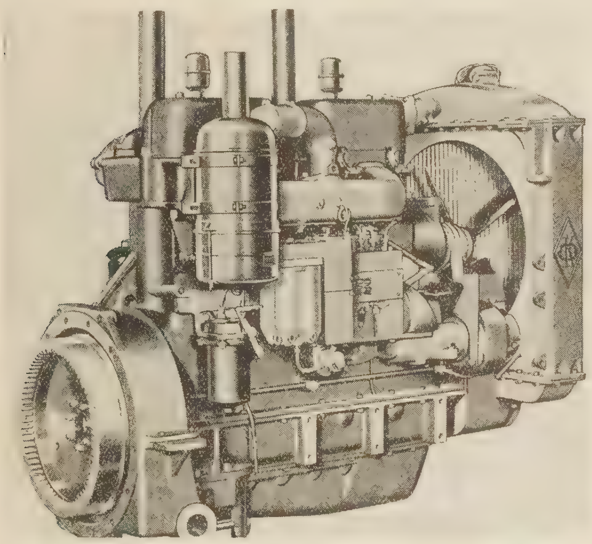

Fig. 4 The Allis-Chalmers Oil Engine

pump so that it will supply the proper quantity of fuel and thus maintain a substantially constant air-fuel ratio.

Electric current for the spark plugs is supplied from either a magneto or from a battery and the customary coil and distributor.

## Construction of the Engine

The Allis-Chalmers oil engine is built with $5\frac{1}{4} \times 6\frac{1}{2}$-in. cylinder dimensions. Its full-load speed is 1050 rpm and the maximum governed speed is 1200 rpm.

The combustion chamber is contained in the cup-shaped piston. The cylinder head is flat. Piston design follows the generally accepted gasoline-engine practice. The pistons are made of tin-plated cast iron. Three compression rings, $\frac{3}{16}$ in. wide, and one oil ring $\frac{1}{4}$ in. wide are employed. Rings are of the same material and dimensions as used in the Allis-Chalmers gasoline engines. The replaceable cylinder liners are made of nickel cast iron and with the customary hardness. Oil and gasoline engines use the same liners.

The cylinder-head design follows standard Diesel practice. The intake and exhaust valves are as large as the cylinder bores permit. The valve ports were designed to give the least possible resistance to the flow of air and gas. For this reason the intake port is streamlined. Air flows from the intake manifold through the port and past the valves into the cylinder in an uninterrupted stream free from eddys or sudden changes in velocity due to the proper cross-sectional variation. Since the intake and exhaust valves are located at either side of the cylinder, air will naturally enter tangentially and the resultant turbulence created thereby is sufficient for maintaining good combustion at high-speed full-load operation.

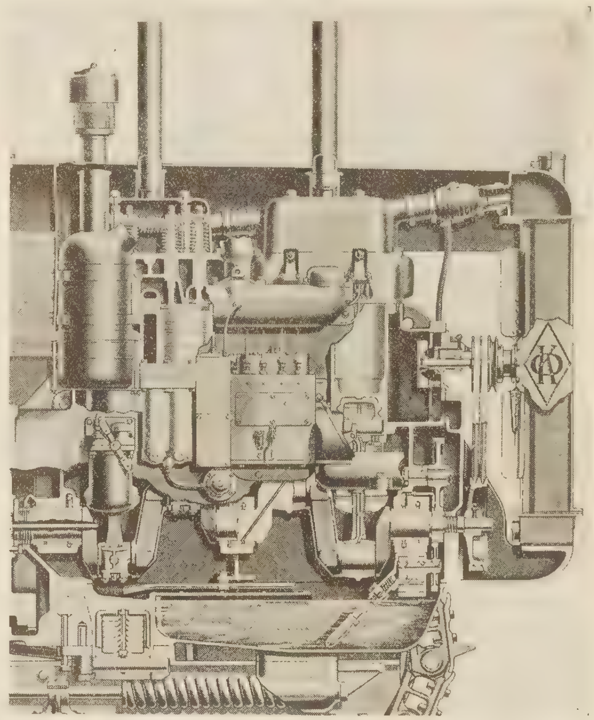

Fig. 5 Sectional View of the Allis-Chalmers Oil Engine

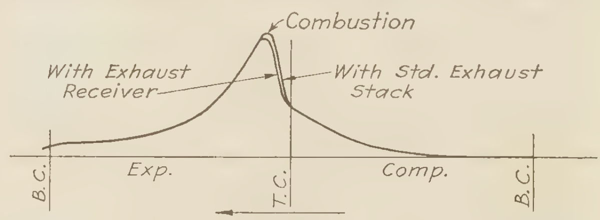


Fig. 6 Influence of Exhaust-Gas Dilution on Combustion Rate

(Curves plotted from results of two runs, one with the engine exhausting into a receiver and one with the engine exhausting into a standard exhaust stack. Both runs were made at 1040 rpm, and with a fuel rate of 2360 revolutions per 2 lb fuel. Note: The residual exhaust gases cause slower combustion and also a loss of power.)

The same streamlining principle was employed for the exhaust ports in order to obtain the maximum evacuation of the cylinders. Exhaust-gas dilution slows down combustion and, therefore, in the low-compression oil engine, it is very important that the combustion chamber, at the end of the exhaust stroke, be filled with exhaust gases of not more than atmospheric pressure, and preferably less. Fig. 6 shows the influence of exhaust-gas dilution on combustion rate.

Great care has been taken in providing large water passages over the combustion chamber and around the valves to prevent hot spots and to maintain uniform combustion temperature in every part of the chamber.

Valves, valve gearing, crankshaft, connecting-rods, and bearings are identical in both the Allis-Chalmers oil and gasoline engines. The fact is both engines are built on the same assembly line. In the following paragraphs is a description of the parts and equipment which are peculiar to the oil engine and not employed on the gasoline engine.

### The Fuel-Injection Equipment

The injection system comprises two main divisions: (1) The injection pump, and (2) the injector. The pump meters the fuel

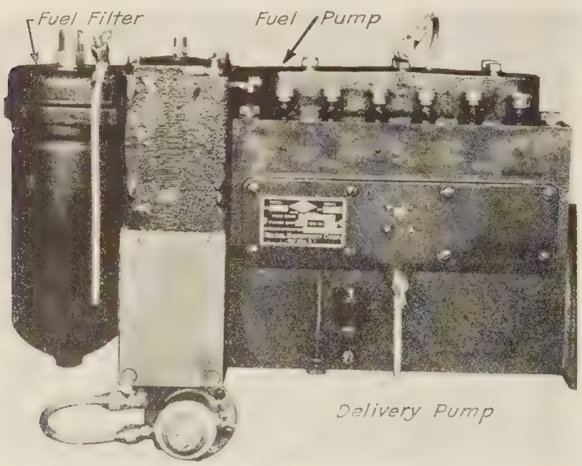


Fig. 7 Deco Pump Assembly Used on the A-C Oil Engine

and delivers it under pressure to the injectors. The injector discharges the fuel into the combustion chamber in a form in which it can be readily burned when suitably heated, mixed with air, and ignited with a spark. The fuel pump used on Allis-Chalmers oil engines is known as the Deco pump. This pump is shown in Fig. 7. The operation of the pump is simple and dependable because it follows the familiar time-proved principle of plunger-and-valve operation.

Fig. 8 shows the simple construction of the Deco pump. The housing, body, and suction-valve covers make up the three sections of the pump. The housing is made of cast iron, with a finished top surface, making a perfect fit between the housing and the body. A machined-steel body is fastened to the pump housing by means of two studs. Capscrews fasten the suction-valve cover, which also serves as a fuel manifold to the top of the body.

The Deco fuel pump includes an individual pump unit for each engine cylinder. Each pump unit consists of a suction valve, plunger and barrel, discharge, and tappet assembly. The barrel fits securely in the body as shown in Fig. 9. Directly above the barrel is the suction-valve body. The lower portion of the valve body is accurately ground as a valve seat, and the upper portion acts as a guide for the suction valve. A small spring insures a quick closure of the valves when the suction force is released. Several small holes drilled in the suction-valve body, just above the seat, permit fuel to pass from the fuel manifold to the valve. The entire valve assembly is held securely in position by the sleeve nut which is screwed into the pump head.

The plunger and barrel, which are located directly beneath the suction-valve assembly, are accurately ground to secure a perfect fit of the plungers in the barrels. The lower end of the plunger is in contact with the tappet adjusting screw located in the upper end of the tappet body assembly. The lower end of the tappet assembly is provided with a roller which engages the camshaft, and the entire tappet assembly is installed with a sliding fit in the pump housing and rests on the flat cross section of the control rod. It is held in place by the heavy plunger spring.

The control rod which extends the entire length of the pump housing passes through all of the tappet bodies between the rollers and tappet screws. The section of the control rod within the tappet bodies is machined flat and by its rotation the plunger stroke and, therefore, the amount of the fuel handled by the pump is controlled.

The pump camshaft running in a bath of oil is supported on both ends by roller bearings with suitable oil seals, and mounted in a chamber which is entirely sealed from the rest of the housing except for a small air vent. In this design, fuel oil which may collect in the upper portion of the housing cannot enter the camshaft chamber and dilute the lubricating oil. A drain is provided for any fuel oil that may collect in the upper portion of the housing.

In front of the suction-valve cover, and located in the body proper, is a discharge valve, consisting of the valve, barrel, and

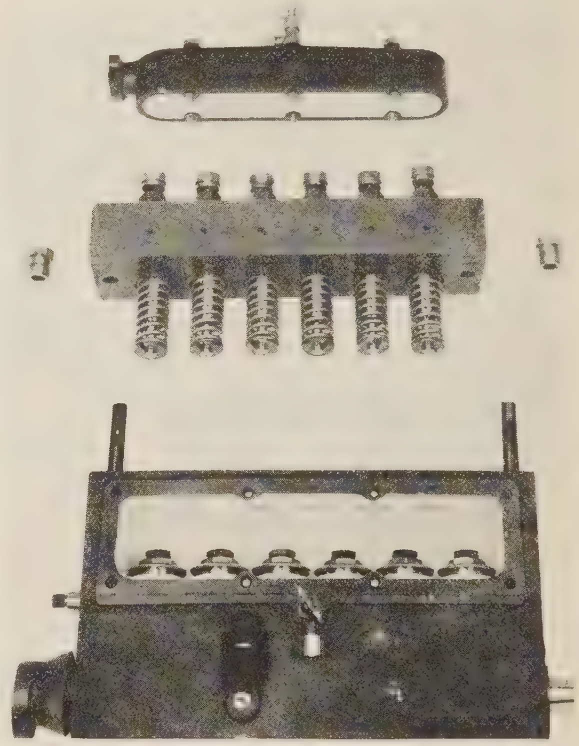

Fig. 8 Construction of the Deco Pump

spring. The valve is drilled to accommodate the spring. The tip of the valve is conical in shape and fits into the lower portion of the barrel which is accurately ground to a valve seat. The upper portion of the barrel serves as a guide for the valve. There are holes drilled in the conical tip of the valve just above the valve seat, and flats are machined along the entire length of the outside of the valve body. By means of these holes and flats, fuel can flow unhindered through the injector tubing and to the injector when the valve is opened.

The Deco pump operates as follows: When the plunger moves downward toward its low position, it creates a suction at suction valve and opens the valve, thus filling the barrel with fuel. When the plunger reaches its lowest position, it remains stationary for

a moment, thus relieving the suction, and the spring closes the suction valve before the plunger starts upward. As the plunger starts upward on its delivery stroke, the fuel within the barrel and in the fuel passage is under a pressure of 1350 lb per sq in., which lifts the discharge valve. Fuel then flows into the injection line to the injector. The injector line carries the fuel to the spray nozzle located in the cylinder head of the engine through which it is sprayed into the combustion chamber.

The flow of fuel into the combustion chamber will continue as long as the pressure remains above 1350 lb per sq in. As soon as the plunger reaches the end of its upward stroke, this pressure falls below 1350 lb per sq in. and the injector closes, terminating injection. The amount of fuel injected by the pump is metered by varying the length of the plunger stroke; the arrangement is shown in Fig. 10.

Rotation of the control rod a few degrees causes the high edge of the control rod to act on the adjusting screw and thus raise the entire tappet assembly. The effect of this movement is to decrease the stroke of the plunger, because the roller will be raised and therefore will be contacted by the cam at a higher point on the cam. By rotating the control rod to the maximum, the roller in the tappet assembly may be raised so that it will never be contacted by the cam. In this position the pump becomes ineffective, which serves to stop the injection.

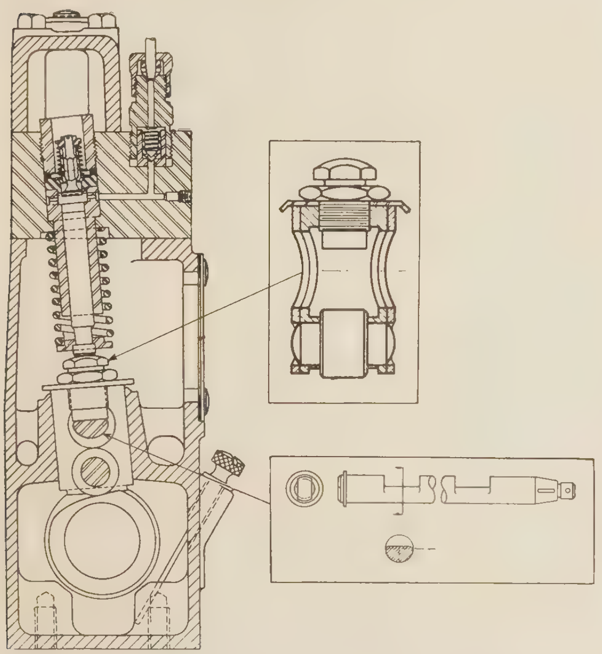

Fig. 9 Sectional View of the Deco Pump Showing the Suction Valve, Plunger and Barrel, and the Tappet Assembly

The second part of the injection system is the injector, the assembly of which is shown in Fig. 11. It contains a spring-loaded nozzle valve that regulates the injection pressure, and is operated hydraulically by the fuel pump. The part of the injector which enters into the combustion chamber is called the tip. The design of the tip determines the shape of the fuel spray, its power of penetration, and the degree in which it is broken up into a fine mist.

A conventional diaphragm pump driven off the fuel-pump camshaft supplies the injection pump with fuel from the tank. This primary pump forces the fuel through a filter before it can enter the injection pump. The filter is a commercial product and capable of removing all particles larger than 0.0015 in. from the fuel. A prefilter installed in the fuel tank removes large particles from the fuel oil.

It was mentioned at the beginning of this paper that not only the fuel but also the air must be governed in accordance with the momentary load on the engine. The engine is throttled and

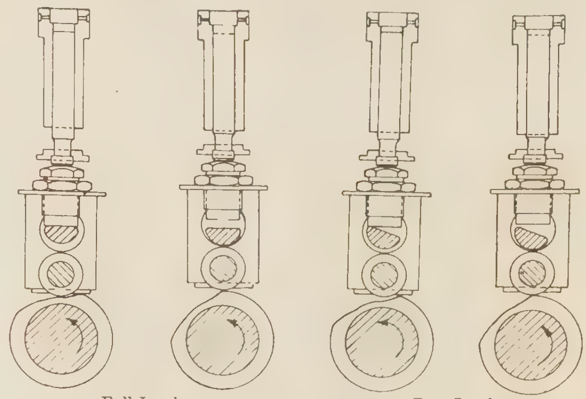


Fig. 10 Showing Variable Stroke of the Deco Pump

governed by means of a vacuum control which is mounted on one end of the fuel-injection pump. The control, shown in Fig. 12, has a vertical spring-loaded piston within a cylinder which, by means of suitable linkage, turns the pump control rod in the fuel pump.

The vacuum control cylinder is airtight except for two outlets, one on each side of the piston. The outlet on the spring-loaded side of the piston is piped to the vacuum side of the butterfly valve in the intake manifold. The other outlet connects to the air-cleaner side of this valve. Any increase in the manifold vacuum will move the piston downward against the spring tension, and any decrease in the manifold vacuum will allow the spring to move the piston upward. The movement of this piston is transferred to the control rod in the pump and causes it to rotate, thus increasing or decreasing the length of the stroke of pump plunger in accordance with the manifold vacuum.

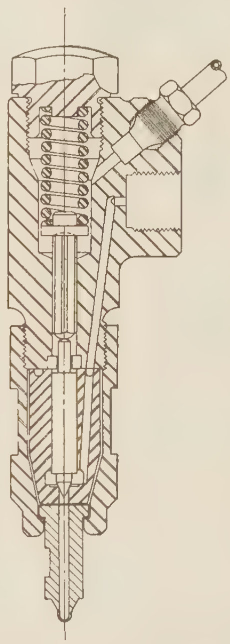

Fig. 11 Deco Injector Assembly

The vacuum in the manifold is controlled by the position of the butterfly valve, which is in turn controlled by a conventional governor.

In the foregoing manner the correct air-fuel ratio is maintained at all loads and speeds; the spring ratio and the vacuum control are so determined as to maintain a definite relationship between the quantity of air drawn into the engine and the fuel injected into the engine, both thus corresponding to the vacuum in the manifold.

Fig. 13, which is shown on the following page, shows the relation of fuel to manifold vacuum, thus indicating the delivery limits of the Deco pump.

### Ignition Equipment

The ignition equipment consists of either a magneto and spark plugs, or a battery with distributor, coil, and spark plugs.

The oil engine requires a hotter spark than the gasoline engine, and therefore the conventional magneto did not give satisfactory service, or rather, it did not have enough reserve capacity for all emergencies. The magneto used for oil engines has a rotor made either of cobalt steel or the so-called alnico alloy, otherwise its design is identical with the conventional types.

Similarly the battery-ignition system had to be revamped to give hotter sparks. Today, both magneto and battery ignition suitable for oil engines are commercially available.

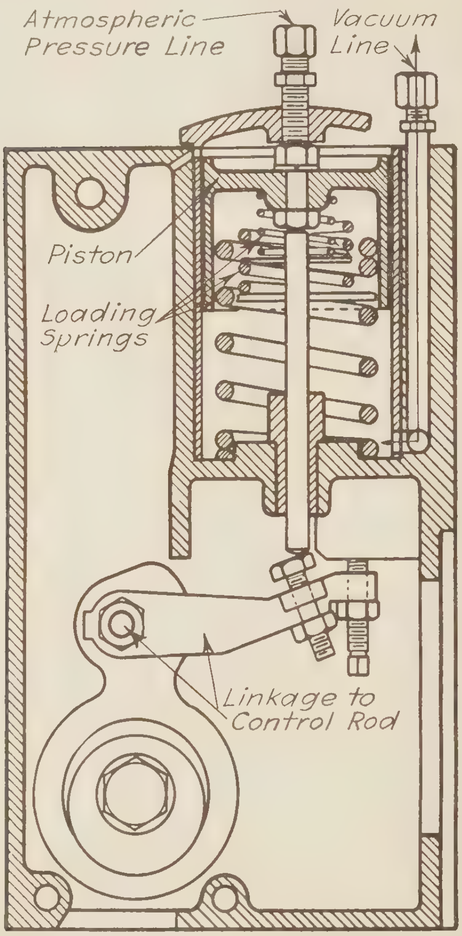


Fig. 12 Vacuum Control for Throttle-Governing the A-C Oil Engine

The spark plugs in the oil engine must show a very high resistance against fouling. Every time an injection takes place the plug is soaked with fuel and it still must not fail to function. It took two years of close cooperative work with the plug manufacturers before the spark-plug problem could be considered as solved. Today there are spark plugs of several makes which will give several hundred hours of uninterrupted service.

### Starting

The Allis-Chalmers oil engine is an easy starting engine. A little gasoline injected into the intake manifold by means of a primer provided for the purpose, two or three upward pulls on the starting crank, and the engine is running. This starting method has been effective at all temperatures.

Due to the low compression pressure there is no necessity to relieve compression or convert the engine for starting into a gasoline engine. There is no fuel to switch over. The fuel oil is injected as soon as the engine starts turning over. The oil engine does not require any more priming or warming up than the gasoline engine.

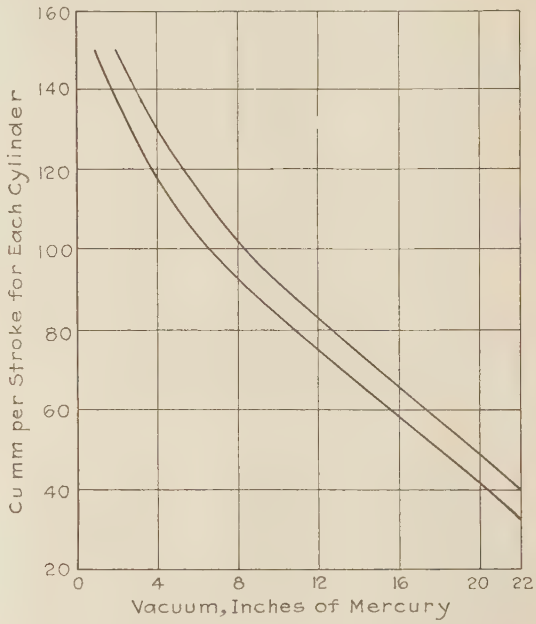


Fig. 13 Delivery Limits of the Deco Injection Pump

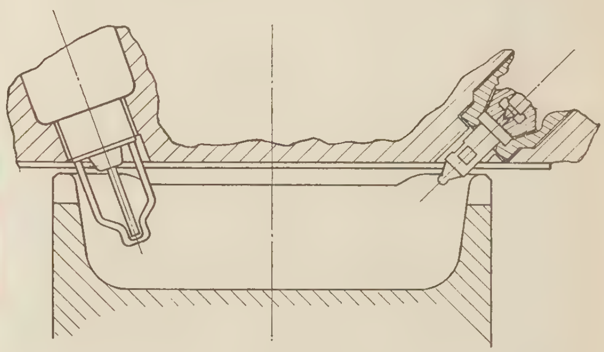

Fig. 14 Combustion Chamber of the A-C Oil Engine

If desired, a 12-v electric starter of conventional capacity and design can be supplied.

### Combustion

Fuel is injected into the combustion chamber by the injector through a multiple-orifice nozzle tip. The number of sprays and their included angle have been so selected that the entire chamber is uniformly supplied with fuel. The sprays are directed toward the bottom of the cup-shaped combustion chamber and at no time during the injection period can they penetrate over the edge of the piston. This arrangement prevents crankcase dilution and at the same time is very beneficial for the rapid vaporization of the fuel. The spark plug penetrates deeply into the combustion chamber and ignites that part of the fuel charge which

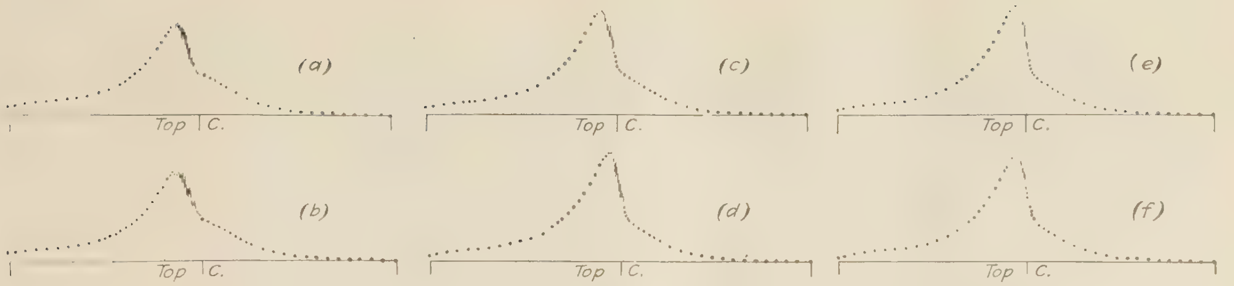


FIG. 15 INDICATOR CARDS TAKEN ON THE A-C OIL ENGINE WHILE RUNNING COLD

| Card | *a* | *b* | *c* | *d* | *e* | *f* |
|---|---|---|---|---|---|---|
| Test no. K.O.—20,002 | 13 | 13 | 13 | 13 | 13 | 13 |
| Cylinder no. | 1 | 1 | 1 | 1 | 1 | 1 |
| Speed, rpm | 1055 | 964 | 830 | 720 | 625 | 510 |
| Beam load lb | 202 | 203 | 205 | 203 | 198 | 193 |
| Rev. per 2 lb of fuel | 3425 | 3495 | 3481 | 3472 | 3471 | 3508 |
| Exhaust gas temp, F | 1115 | 1020 | 920 | 860 | 830 | 760 |
| Jacket water temp, F | 132 | 128 | 124 | 124 | 122 | 120 |
| Exhaust condition | I | I | I | I | Haze | Smoke |

I = invisible; L.P. = light ping; P = Ping.

| Card | *a* | *b* | *c* | *d* | *e* | |
|---|---|---|---|---|---|---|
| Ignition: | | | | | | |
| Cyl. no. 1 | Spark | Spark | Self | 50/50 | 50/50 | Spark |
| Cyl. no. 2 | Spark | 50/50 | Self | Self | 50/50 | Spark |
| Cyl. no. 3 | 50/50 | Self | Self | Self | 50/50 | 50/50 |
| Cyl. no. 4 | 50/50 | Self | Self | Self | 50/50 | 50/50 |
| Running knocks | L.P. | L.P. | P | P | P | P |
| Indicator spring scale, lb per in. | 300 | 300 | 300 | 300 | 300 | 300 |
| Pump timing, deg | 58 | 58 | 58 | 58 | 58 | 58 |
| Magneto timing, deg | 15 | 15 | 15 | 15 | 15 | 15 |
| Manifold vacuum, in. Hg | 1.25 | 1.1875 | 0.875 | 0.75 | 0.625 | 0.4375 |

is already in vapor state. A cross section of the combustion chamber is shown in Fig. 14.

All tests with various spark-plug locations, injection, and spark timing, combustion-chamber temperatures, nozzle designs, and turbulence, showed that in a spark-ignition fuel-injection oil engine, ignition of the fuel takes place from the vapor state. This contention is supported by the fact that the combustion-chamber temperature bears a very definite relation to the start of the combustion. Indicator cards taken with the Prescott indicator show clearly that this is the case.

It can be seen from the cards in Fig. 15, which were taken with a cold engine, that the combustion line is very irregular even at slower speeds when the engine is running with self-ignition. These cards should be compared with those taken with a warmer combustion chamber as shown in Fig. 16.

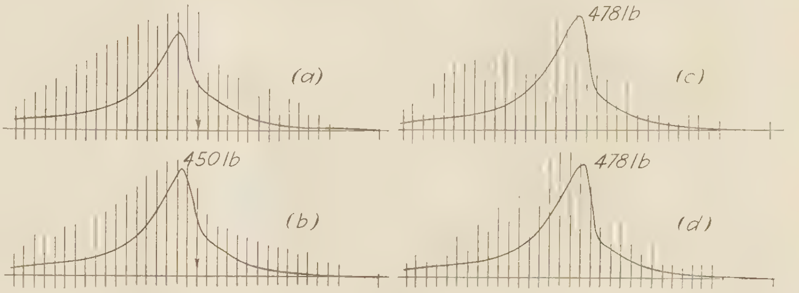


FIG. 16 INDICATOR CARDS TAKEN ON THE A-C OIL ENGINE WHILE RUNNING WARM

| Card | *a* | *b* | *c* | *d* |
|---|---|---|---|---|
| Test no. | 12-1 | 12-1 | 12-1 | 12-1 |
| Cylinder | 4 | 4 | 4 | 4 |
| Speed, rpm | 1100 | 925 | 725 | 629 |
| Beam load, lb | 197 | 207 | 190 | 195 |
| Rev. per 2 lb of fuel | 3514 | 3516 | 3441 | 3568 |
| Exhaust-gas temp, F. | 1200 | 1100 | 1020 | 1000 |
| Jacket-water temp, F. | 138 | 139 | 145 | 120 |
| Exhaust condition | I | I | Haze | Haze |
| Ignition | Spark | 50/50 | Self | Self |
| Running knocks | ... | L.P. | H.P. | H.P. |
| Indicator spring scale, lb per in | 300 | 300 | 300 | 300 |
| Pump timing, deg | 58 | 58 | 58 | 58 |
| Magneto timing, deg | 15 | 15 | 15 | 15 |
| Manifold vacuum, in. hg | 1.875 | 1 | 0.6875 | 0.625 |

L.P. = light ping; H.P. = heavy ping; I = invisible.

Much can be learned about combustion by the use of indicator cards that show the relation of cylinder pressure and angular position of the crankshaft with the *P-T* cards. Cards shown in this paper were taken with a Prescott indicator which was found to give very reliable readings.

Fig. 17 shows five cards taken during a torque-curve run; they represent typical full-torque performance of the engine with standard equipment and setting.

Information obtained by analysis of indicator cards was plotted for various tests. The time rate of pressure rise shown in Figs. 18 and 19 is very interesting. In general it will be found that the maximum time rate of pressure rise or corresponding rate of burning occurs at some speed intermediate between the low-speed and high-speed limits in an Allis-Chalmers oil engine. This maximum time rate of pressure rise or combustion occurs at about 750 to 800 rpm but may vary from 600 to 800 rpm with various setups of apparatus. In a smoothly operating engine this time rate of maximum pressure rise will reach a value of from 100,000 to 150,000 lb per sq in. per sec.

Fig. 18 shows tests results of a setup with a special cam in the injection pump giving an increasing rate of fuel discharge. The time rate of maximum pressure rise clearly shows the influence of discharge rate upon combustion.

Fig. 19 shows test results of the standard setup of the engine. Again the time rate of maximum pressure rise depicts results with a constant rate of fuel discharge during injection.

In order to burn the fuel efficiently, a rapid and well-timed combustion must be attained. The fuel must be burned as near to top dead center as possible to give a maximum expansion of the gases. Thus, the actual start of combustion must start before top dead center so that the maximum pressure rise will come near top center. The mechanical strength of the engine and the smoothness of engine operation modify the above theoretical considerations such that a maximum pressure of 30 lb per sq in. per deg of crankshaft rotation is allowable; also the maximum pressure must not occur sooner than 10 deg after top center. If the foregoing limits are exceeded, there will be excessive pinging with consequent loss of power. Thus the pressure rise per degree and the angle of maximum pressure after top center are knock factors; see Fig. 19.

It has already been mentioned that vaporization of fuel is an important factor in engine performance and combustion. Fig.

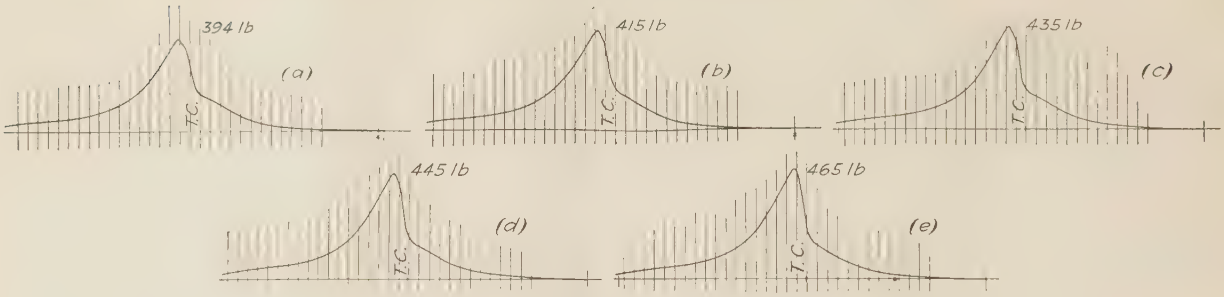


Fig. 17 Full-Torque Indicator Cards Taken on the A-C Oil Engine

| Card | a | b | c | d | e |
|---|---|---|---|---|---|
| Test no. | 14 | 14 | 14 | 14 | 14 |
| Cylinder no. | 1 | 1 | 1 | 1 | 1 |
| Speed, rpm | 1051 | 908 | 750 | 612 | 456 |
| Beam load, lb | 292.4 | 300 | 307 | 311.5 | 301 |
| Rev. per 2 lb of fuel | 2504 | 2537 | 2538 | 2540 | 2549 |
| Exhaust-gas temp, F | 1290 | 1240 | 1130 | 1020 | 900 |
| Jacket-water temp, F | 138 | 152 | 140 | 138 | 146 |

| Card | a | b | c | d | e |
|---|---|---|---|---|---|
| Exhaust condition | I | I | I | S.S. | Smoke |
| Ignition | 50/50 | 50/50 | 50/50 | Self | Self |
| Running knocks | ... | .... | ... | .... | .... |
| Indicator spring scale, lb per in. | 300 | 300 | 300 | 300 | 300 |
| Pump timing, deg | 60 | 60 | 60 | 60 | 60 |
| Magneto timing, deg | 12 | 12 | 12 | 12 | 12 |
| Manifold vacuum, in. Hg | 1.375 | 1.0625 | 0.75 | 0.4375 | 0.3125 |
| Air temp, F | 86 | 86 | 84 | 78 | 82 |

I = invisible, S.S. = slight smoke.

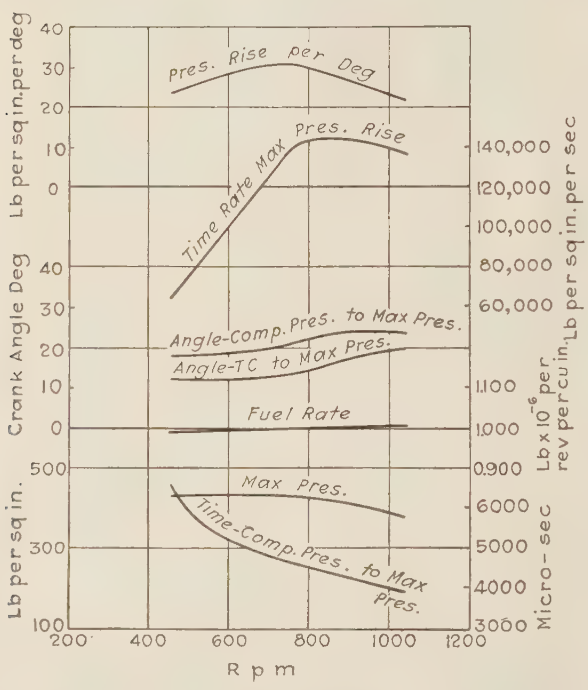


Fig. 18 Results Obtained With a Special Cam in the Injection Pump to Give an Increased Rate of Fuel Discharge

(Results for No. 6 cylinder with 60-deg pump timing and 12-deg spark timing.)

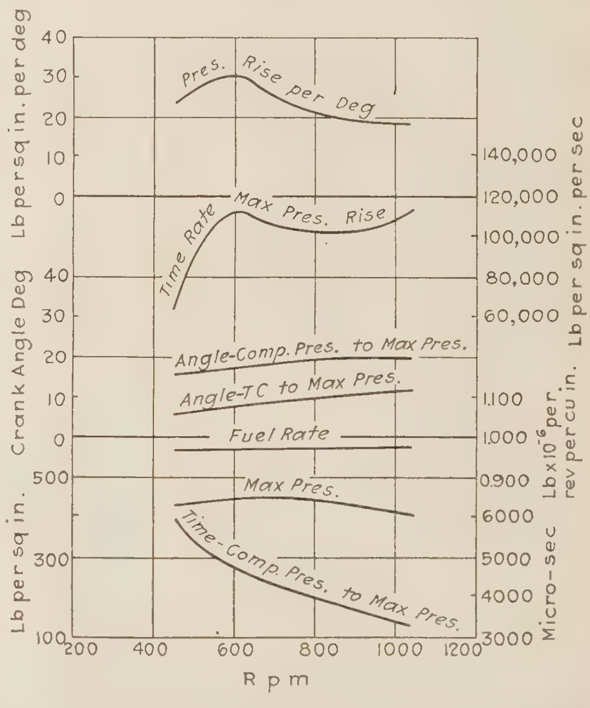


Fig. 19 Results Obtained With a Standard Cam Arrangement in the Injection Pump

(Results for No. 6 cylinder with 60-deg pump timing and 12-deg spark timing.)

20 gives a vivid picture of the influence of pump timing or influence of time element on the rate of combustion. Both runs shown on each of the cards in Fig. 20 were made under practically identical conditions in so far as speed, fuel rate, water temperature, and manifold vacuum were concerned. The main difference between the two runs lies in the pump timing of 53 deg and 58 deg, respectively. The variation in the spark timing during the two runs did not affect the engine performance.

The timing of the spark plays a very small rôle in the rate of combustion. For the most efficient performance in the present A-C oil engine, there must be a time element of about 45-deg crankshaft angle between the start of the injection and the occurrence of the spark at full load. Reducing this interval brings skipping, and increasing the interval causes afterburning, but never skipping. The fact is that any engine can be made to perform by simply retarding the spark. Thus, if an injector were not operating properly and the fuel is not being distributed or atomized, a longer period of time is required for the ignitible mixture to form. This would seem to be a function of the time required for vaporization of the fuel, since vaporization is a time function.

Turbulence was found to be necessary for high-speed full-load operation with the open-type combustion chamber and with the spark located opposite the injector. We have good reason to believe that in the Allis-Chalmers oil engine, the chief benefit derived from the turbulence within the combustion chamber is to effect uniform mixing throughout the entire chamber by

continuous agitation and movement of the mixture which facilitates ignition and increases the rate of flame propagation. However, no amount of turbulence will make up for a partially plugged nozzle tip, and the engine will skip until the injector is repaired. It is believed that in an absolutely uniform mixture, turbulence will not be necessary. This is impossible to attain in practice, but as it was approached, performance of the engine was enhanced in the same degree. At speeds below 1000 rpm and part-load operation, turbulence was found to be not only unnecessary but to be very undesirable.

The following figures present data relating to performance of the engine:

Fig. 21 shows the part-load performance when engine is set rich. Fig. 22 shows a set of indicator cards taken at various loads of the engine. Fig. 23 shows a set of weak-spring cards taken at variable speeds and fully opened throttle. Note good evacuation of exhaust gases. Fig. 24 shows a set of full-load cards taken during a torque-curve run. Figs. 25 and 26 show the full-load performance curves, both observed and corrected, of the fully equipped engine with fan, radiator, air cleaner, water pump, and generator. A standard set of apparatus was used. The fuel oil was Stanolex No. 3. The lowest fuel consumption was 0.440 lb per bhp-hr and the maximum indicated mep was 115 lb per sq in.

The engine runs at all speeds and loads with invisible exhaust. At very low-speed lugging a slight color in the exhaust appears. The engine will idle as low as 160 rpm consistently. There is no crankcase dilution, no deposit of carbon on the pistons or on the valves.

Engine will operate on gasoline, kerosene, distillates, and commercial Diesel oil or domestic furnace oils, as long as the fuel will flow at surrounding temperature. Gasoline, kerosene, and distillates when used are to be mixed with 3 to 5 per cent engine lubricating oil which is required for lubrication of the injection apparatus.

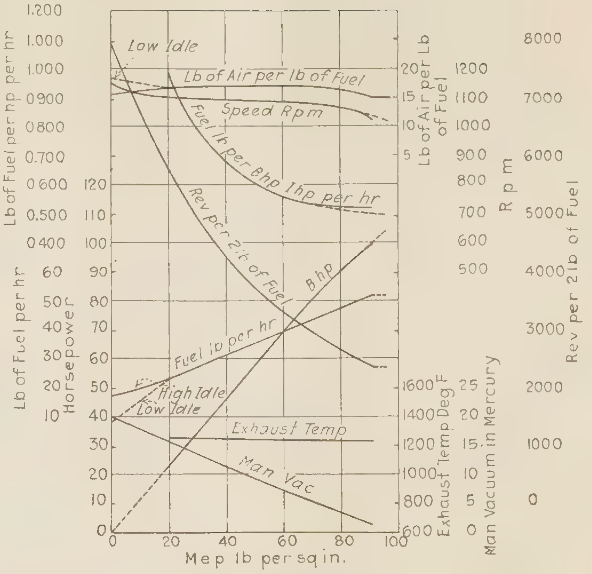


FIG. 21 PERFORMANCE CHARACTERISTICS OF THE A-C OIL ENGINE WITH RICH FUEL INJECTION

The development of this oil engine consumed two years of research work, but the author believes that only a start has been made with this new-type oil engine. Much research work still remains to be done before the fuel-injection spark-ignition engine will assume its proper place among internal-combustion engines. The engine described in this paper is now in regular production and is used in crawler-type tractors and for power units.

[Figs. 22, 23, 24, 25 and 26 appear on the pages immediately following.]

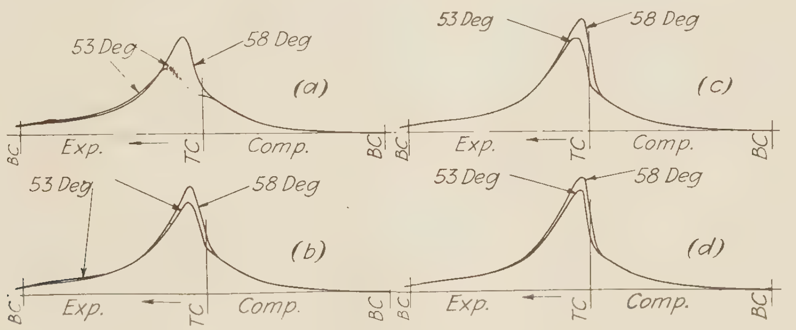


FIG. 20 INDICATOR CARDS SHOWING THE INFLUENCE OF PUMP TIMING ON THE RATE OF COMBUSTION

| Pump timing | 58 deg | | | | 53 deg | | | |
|---|---|---|---|---|---|---|---|---|
| Card | *a* | *b* | *c* | *d* | *a* | *b* | *c* | *d* |
| Test no | 12-1 | 12-1 | 12-1 | 12-1 | 12-1 | 12-1 | 12-1 | 12-1 |
| Cyl. no | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| Speed, rpm | 1100 | 925 | 629 | 725 | 1060 | 944 | 774 | 640 |
| Beam load, lb | 197 | 207 | 195 | 190 | 186.5 | 201 | 200 | 197 |
| Rev. per 2 lb of fuel | 3514 | 3516 | 3568 | 3441 | 3483 | 3466 | 3478 | 3530 |
| Exhaust-gas temp, F | 1200 | 1100 | 1000 | 1020 | 1280 | 1170 | 1100 | 1020 |
| Jacket-water temp, F | 138 | 139 | 120 | 145 | 140 | 142 | 140 | 136 |
| Exhaust condition | I | I | Haze | Haze | L.H. | L.H. | L.H. | L.H. |
| Ignition | S | 50/50 | Self | Self | S[1] | S | Self | Self |
| Running knocks | ... | L.P. | Ping | Ping | ... | .. | Ping | H.P. |
| Indicator-spring scale, lb per in | 300 | 300 | 300 | 300 | 300 | 300 | 300 | 300 |
| Magneto timing, deg | 15 | 15 | 15 | 15 | 10 | 10 | 10 | 10 |
| Manifold vacuum, in. Hg | 1.187 | 1 | 0.625 | 0.687 | 1.125 | 1 | 0.687 | 0.562 |

I = invisible; S = spark; L.H. = light haze; L.P. = light ping; H.P. = heavy ping.
[1] Occasional miss.

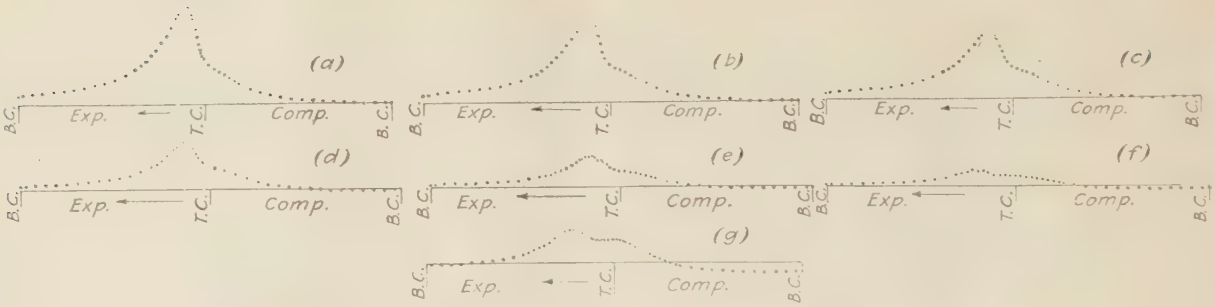


FIG. 22 INDICATOR CARDS TAKEN ON THE A-C OIL ENGINE AT VARIOUS LOADS

| Card | a | b | c | d | e | f | g |
|---|---|---|---|---|---|---|---|
| Test no. | 13-1 | 13-1 | 13-1 | 13-1 | 13-1 | 13-1 | 13-1 |
| Cyl. no. | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| Speed, rpm | 1040 | 1075 | 1081 | 1125 | 1145 | 1160 | 470 |
| Beam load, lb | 296.5 | 254 | 204.5 | 116.6 | 56 | Idle | Idle |
| Rev. per 2 lb of fuel | 2366 | 2778 | 3240 | 4345 | 5641 | 7502 | 9466 |
| Exhaust-gas temp, F | 1360 | 1410 | 1410 | 1460 | 1460 | 1360 | 800 |
| Jacket-water temp, F | 154 | 146 | 140 | 150 | 142 | 142 | 174 |

| Card | a | b | c | d | e | | g |
|---|---|---|---|---|---|---|---|
| Exhaust condition | I | I | I | I | I | I | I |
| Ignition | S | S | ... | ... | ... | . | . |
| Indicator-spring scale, lb per in. | 300 | 300 | 300 | 300 | 300 | 300 | 80 |
| Pump timing, deg | 60 | 60 | 60 | 60 | 60 | 60 | 60 |
| Magneto timing, deg | 12 | 12 | 12 | 12 | 12 | 12 | 12 |
| Manifold vacuum, in. Hg | 1.375 | 4 | 6.625 | 12.125 | 16.375 | 20 | 21.75 |

I = invisible; S = spark.

NOTE: When these cards were taken the injection pump was fitted with a special cam to give an increased rate of fuel discharge, see Fig. 18.

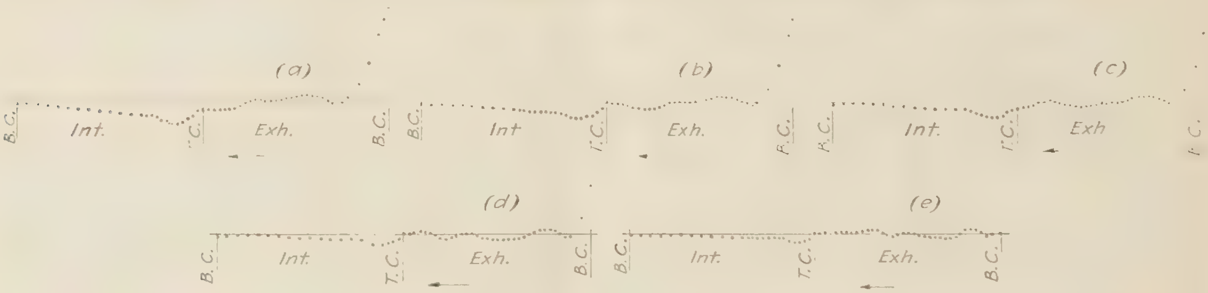


FIG. 23 WEAK-SPRING INDICATOR CARDS TAKEN ON THE A-C OIL ENGINE AT VARIOUS SPEEDS AND WIDE-OPEN THROTTLE

| Card | a | b | c | d | e |
|---|---|---|---|---|---|
| Test no. | 14 | 14 | 14 | 14 | 14 |
| Cylinder no. | 1 | 11 | 1 | 1 | 1 |
| Speed, rpm | 1050 | 905 | 770 | 602 | 450 |
| Beam load, lb | 295 | 302.2 | 309.5 | 309.5 | 302.3 |
| Rev. per 2 lb fuel | 2467 | 2518 | 2534 | 2553 | 2563 |
| Exhaust-gas temp, F | 1280 | 1220 | 1125 | 1010 | 875 |
| Jacket-water temp, F | 138 | 136 | 138 | 135 | 138 |

| Card | a | b | c | d | e |
|---|---|---|---|---|---|
| Exhaust condition | I | I | I | S.S. | S |
| Air temp, F | 82 | 86 | 82 | 80 | 82 |
| Indicator-spring scale, lb per in. | 16 | 16 | 16 | 16 | 16 |
| Pump timing, deg | 60 | 60 | 60 | 60 | 60 |
| Magneto timing, deg | 12 | 12 | 12 | 12 | 12 |
| Manifold vacuum, in. Hg | 1.5 | 1.125 | 0.75 | 0.5 | 0.3125 |

I = invisible; S.S. = slight smoke; S = smoke.

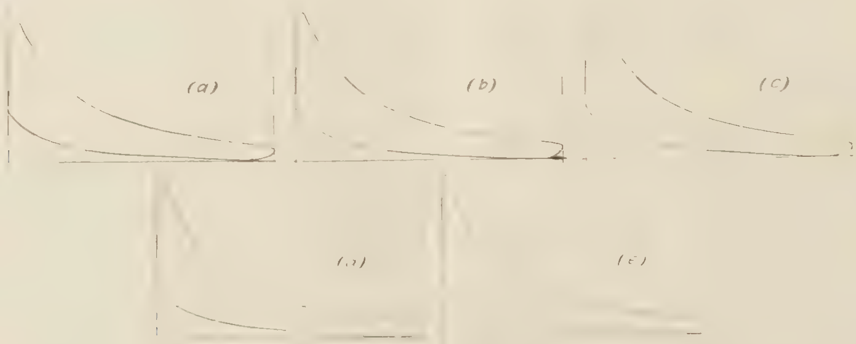


FIG. 24 FULL-LOAD INDICATOR CARDS TAKEN ON THE A-C OIL ENGINE DURING A TORQUE-CURVE RUN

| Card | a | b | c | d | e |
|---|---|---|---|---|---|
| Test no. | 14 | 14 | 14 | 14 | 14 |
| Cylinder no. | 1 | 1 | 1 | 1 | 1 |
| Speed, rpm | 1051 | 908 | 750 | 612 | 456 |
| Beam load, lb | 292.4 | 300 | 307 | 311.5 | 301 |
| Rev. per 2 lb fuel | 2504 | 2537 | 2538 | 2540 | 2549 |
| Exhaust-gas temp, F | 1290 | 1240 | 1130 | 1020 | 900 |
| Jacket-water temp, F | 138 | 152 | 140 | 138 | 146 |
| Exhaust condition | I | I | I | S.S. | S |
| Ignition | 50/50 | 50/50 | 50/50 | Self | Self |

| Card | a | b | c | d | e |
|---|---|---|---|---|---|
| Indicator-spring scale, lb per in. | 100 | 100 | 100 | 100 | 100 |
| Pump timing, deg | 60 | 60 | 60 | 60 | 60 |
| Magneto timing, deg | 12 | 12 | 12 | 12 | 12 |
| Manifold vacuum, in. Hg | 1.375 | 1.0625 | 0.75 | 0.4375 | 0.3125 |
| Barometric pressure, in. Hg | 29.55 | 29.55 | 29.55 | 29.55 | 29.55 |
| Air temperature, F | 86 | 86 | 84 | 78 | 82 |
| Indicated mep, lb per sq in. | 108 | 110 | 111 | 110 | 108 |
| Corrected indicated mep, lb per sq in.[a] | 112 | 114 | 115 | 113 | 112 |
| Correction factor | 1.037 | 1.037 | 1.036 | 1.03 | 1.034 |

[a] Corrected to a barometric pressure of 29.92 in. Hg, and 60 F.
I = invisible; S.S. = slight smoke; S = smoke.
NOTE: When these cards were taken the injection pump was fitted with a special cam to give an increased rate of fuel discharge, see Fig. 18.

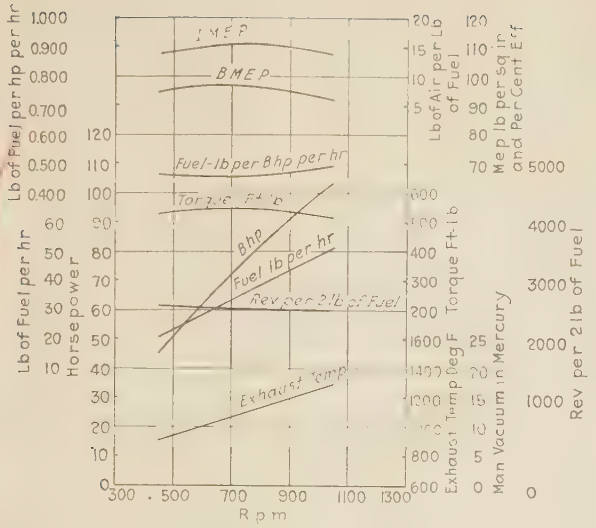


FIG. 25 FULL-LOAD PERFORMANCE CURVES OF THE A-C OIL ENGINE EQUIPPED WITH FAN, AIR CLEANER, WATER PUMPS, AND LIGHTING GENERATOR

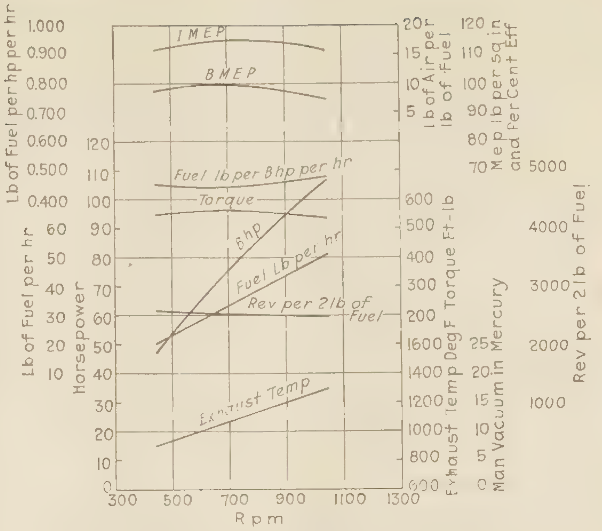


FIG. 26 CORRECTED FULL-LOAD PERFORMANCE CURVES OF THE A-C OIL ENGINE EQUIPPED WITH FAN, AIR CLEANER, WATER PUMPS, AND LIGHTING GENERATOR

# A Review of Existing Psychrometric Data in Relation to Practical Engineering Problems

By W. H. CARRIER,[1] NEWARK, N. J., and C. O. MACKEY,[2] ITHACA, N. Y.

**The authors review and correlate available psychrometric data, and discuss the application of these data to engineering problems. They analyze and correlate existing data with reference to deviations of observed wet-bulb temperatures from those of true adiabatic saturation. The paper also includes a tabulation of revised psychrometric values in accordance with the latest physical data with correction factors for all normal variations of barometric pressures. The authors make an analysis and give a demonstration of the proper method of employing the psychrometric heat function previously defined as the "total heat less the heat of the liquid," and afterward referred to in psychrometry as "total heat." For this function the authors offer the term "sigma function," to distinguish it from the enthalpy or true total heat which includes the heat of the liquid.**

In view of the present wide employment of psychrometric data in various fields, and particularly in the field of air conditioning, and also in view of the fact that there have been numerous questions raised as to the limits of accuracy of existing data, it seems opportune to review and correlate, as far as possible, the past research in this field and to discuss the application of these data to engineering problems.

## Objectives of Paper

*First Objective.* The analysis and correlation of existing data with reference to deviations of observed wet-bulb temperatures from those of true adiabatic saturation.

*Second Objective.* Presentation of a tabulation of revised psychrometric values in accordance with the latest physical data with correction factors for all normal variations of barometric pressures. The purpose of this is to permit the ready use of standard psychrometric data or charts for any other barometric pressure without involved calculations. The latter is a device that has long been needed and is of particular value in accurate calculation of test data.

*Third Objective.* An analysis and a demonstration of the proper method of employing the important and useful psychrometric heat function, previously defined by Carrier in 1911 (1),[3] as the "total heat less the heat of the liquid," and afterward referred to in psychrometry as "total heat." For this function will now be offered the term, the "sigma function" to distinguish it from the enthalpy or true total heat which includes the heat of the liquid.

## Deviation of Observed Wet-Bulb Temperature From Temperature of Adiabatic Saturation

In the past 25 years, most of the engineering calculations involving humidity in air have been based on the psychrometric chart presented in 1911 by Carrier (1). The values given were based, not primarily, as many assume, on observed wet-bulb temperatures with a sling psychrometer, but on calculated values of adiabatic saturation. The paper (1) showed that experimentally the two values were in close agreement. However, test data presented in the original paper (1) in 1911 indicated two sources of deviation of the wet-bulb temperature from the temperature of adiabatic saturation. The first was the radiation factor which was indicated by the difference between the reading of an unshielded wet bulb and a wet bulb completely shielded from radiation, and the second was the difference between the reading of a radiation-shielded wet bulb and the observed temperature of adiabatic saturation, where the radiation-shielded wet bulb apparently gave the lower reading of the two. In view of the theory advanced, the latter variation was, however, thought at the time, to be due to an apparatus error.

Later observations conducted by Arnold (2) and by Dropkin (3) have shown the latter assumption to be incorrect and that it is not only possible, but in accordance with physical laws that the latter variation should exist. However, these two sources of deviation are in opposite directions tending to neutralize each other, and it has been proved that there is a definite air velocity where there is exact agreement between the wet bulb (not shielded from radiation) and the actual temperature of adiabatic saturation. Carrier's 1911 experiments (1) would indicate this velocity to be about 2000 fpm. Dropkin's test (3) would indicate it to be slightly over 1000 fpm. Arnold (2) would fix this velocity at about 500 fpm. Computations from his theory give a still lower value as shown in Appendix 1. The authors' present correlation would indicate it to be at an intermediate velocity

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[1] Chairman of the Board, Carrier Engineering Corporation. Mem. A.S.M.E. Mr. Carrier was graduated from Cornell University in 1901 and upon graduation accepted the position of research engineer with the Buffalo Forge Company. Five years later he became chief engineer. As the science of air conditioning developed under his guidance he saw the necessity for a separate organization and accordingly the Carrier Engineering Corporation was formed in 1915. Mr. Carrier is a member of the American Society of Heating and Ventilating Engineers and a past-president of the American Society of Refrigerating Engineers. He is the author of various scientific papers, among them a paper entitled "Rational Psychrometric Formulae" presenting the theory and practical data on which the art of air conditioning has been founded and in recognition of which he was elected to Sigma Xi in 1914. Mr. Carrier was awarded the A.S.M.E. Medal in 1934 for his work in air conditioning.

[2] Professor of Heat-Power Engineering, Cornell University. Professor Mackey was graduated from Cornell in 1926 with the degree of M.E. and for the next two years served as instructor of experimental engineering at the University. He then was made assistant professor of heat-power engineering and this year received a full professorship. He is the author of articles on psychrometric principles for the American Society of Heating and Ventilating Engineers and the American Society of Refrigerating Engineers. He is a member of the scientific fraternities Sigma Xi and Tau Beta Pi.

Contributed by the Heat Transfer Committee of the Process Industries Division and presented at the Annual Meeting of The American Society of Mechanical Engineers, held in New York, N. Y., November 30 to December 4, 1936.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until February 10, 1937, for publication at a later date. Discussion received after the closing date will be returned.



[3] Numbers in parentheses refer to the Bibliography at the end of the paper.

probably in the neighborhood of 800 to 900 fpm. Assuming the velocity of air over a sling psychrometer to be between 1000 and 1500 fpm, the present correlation, as shown in Fig. 1 and Appendix 2, would give the deviation of the wet-bulb depression as approximately —0.005. In other words, to the observed temperature of the wet-bulb thermometer there should be added approximately 0.5 per cent of the wet-bulb depression to obtain the true temperature of adiabatic saturation for use in the psychrometric chart or tables. Arnold's theory and data, given in Appendix 1, would indicate a greater variation, but actually he did not observe the temperature of adiabatic saturation, but calculated it, while Dropkin made direct observations of this temperature.

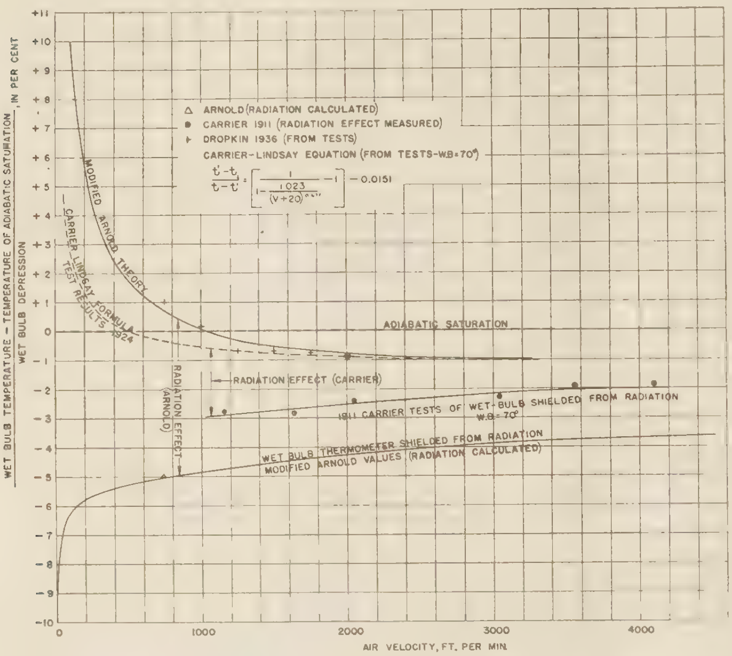


Fig. 1 Correlation of Existing Psychrometric Data With Arnold Theory

In any event, it appears that the best criterion to be employed in psychrometric measurements and the preparation of psychrometric charts and data, is the temperature of adiabatic saturation, because: (1) It is independent of velocity and has an exact physical basis, while the wet-bulb temperature differs from it slightly by amounts varying with velocity. (2) The relation between wet-bulb temperature and that of adiabatic saturation is a basic ratio in Arnold's theory which is the most satisfactory theory so far advanced and which appears to be sound in principle. (3) The exact deviations of the wet-bulb temperature still remain to be determined experimentally. (4) In air-conditioning practice, psychrometric observations are of quite secondary importance to the thermal calculations for which the present type of chart is ideally suited.

Moreover, except for exact research work, the error in substituting the adiabatic-saturation temperature for the observed wet-bulb temperature is negligible, being, generally, a smaller error than that probable in commercial thermometry. For example, in a 20-F observed depression, the deviation probably lies between —0.1 F and 0.2 F which would require the employment of thermometers of extreme precision. In fact, the deviation is less than the variation between two successive accurate thermometer readings, because of the lack of perfect homogeneity of most normal atmospheres. It is for this reason that it has been the general practice heretofore, to employ the psychrometric chart, even in connection with direct psychrometric observations. Although more recent data and theory change the supposed variation from plus to minus, the actual discrepancy is probably no greater than that previously assumed.

Fig. 1 gives the correlation of four independent investigations of the percentage deviation of the wet-bulb depression from that corresponding to the true temperature of adiabatic saturation for different velocities, and also indicates the probable percentage of radiation error. A full discussion of the data employed and the method of their correlation are given in Appendix 2. Here also will be found discussion of the various methods of experimental approach that are available for further research of these important relationships. Attention should be called to the fact that for extremely accurate determinations, the present data are not in sufficient agreement to be wholly trustworthy nor have they been sufficiently explored for wide variations in wet-bulb temperatures; hence, the present correlation is only claimed to be tentative. These data, however, are probably as accurate as is the present assumed value of 0.24 for specific heat of air, which is possibly in error as much as plus or minus 0.5 per cent. Complete psychrometric research will not only determine the relation of the wet-bulb temperature to the temperature of adiabatic saturation and to the moisture content of the air, but will also, perhaps, provide the most reliable method for evaluating the specific heat of air.

## Present Rational Psychrometric Theory

The close experimental agreement of the observed wet-bulb temperature with the temperature of adiabatic saturation is, as stated by Lewis (4) in discussion of the Arnold theory, largely fortuitous, i.e., it holds approximately true for the water-air combination. If an atmosphere were chosen having a somewhat lower molecular weight, as illuminating gas for example, it would hold still more exactly. With combinations of evaporating fluids and atmosphere having wide differences in molecular weights, and therefore in diffusivities, there has been proved by Arnold to be a pronounced discrepancy between the temperature of adiabatic saturation and the wet-bulb temperature.

There were previously two theories extant. The first was that there was a film of saturated air at the temperature of adiabatic saturation surrounding the wet bulb or at the surface of the evaporating liquid. This, at high velocities, was mechanically distributed in constant relative proportions into the surrounding air and the result was in accord with the simple law of mixtures. The second, and opposing theory, was that of Regnault and Maxwell who analyzed the process primarily in accordance with the laws of molecular diffusion, which they assumed to be the controlling process. Carrier arrived at the former theory independently through experimental data obtained in air-conditioning and drying processes, and also by comparing the calculated temperature of adiabatic saturation with the wet-bulb temperatures given in psychrometric tables of the U. S. Weather Bureau.

Carrier and Lindsay (5) recognized the force of the diffusion theory, but in view of the results of Lindsay's experiments, they considered the process of adiabatic saturation to be a limiting case. In other words, the wet-bulb temperature could reach

that of adiabatic saturation but could not go below it. Later observations have shown these data and the assumptions based thereon, to be in error. Dropkin, employing the experimental equipment of Carrier and Lindsay, obtained definitely higher adiabatic saturation temperatures. This discrepancy can be attributed to an inadvertent defect in the equipment as used by Carrier and Lindsay. The supply pipe providing the water to wet the boundary surface had become plugged with dirt so that the process was not adiabatic as assumed. Dropkin corrected this instrumental error and thus obtained different results. Arnold approached the problem from a different angle, using vapors of far lower diffusivity than water vapor, and correlated the deviations observed in a most ingenious and apparently satisfactory manner. His theory is a combination of the diffusion theory with the adiabatic saturation or mechanical mixture theory.

In the evaporative process there are apparently two zones having different temperature and moisture gradients. The first zone is that next to the liquid having purely nonturbulent or laminar flow; the second and outer zone is that in which there is primarily turbulent flow. In the first zone, molecular diffusion controls; in the second zone, mechanical mixture predominates. To give a homely analogy of this effect, assume that a crowd of people are swarming from the street to an elevator system and that a similar crowd are swarming from the elevator in the opposite direction, to the street. The area through which they are passing from the street to the elevator, at their own speed of locomotion, represents the diffusion zone, while the conveyance in the elevator is purely mechanical and the rate of transportation is not affected by their personal activities. The elevator system corresponds to the second zone of mechanical mixture. If, to complete the analogy, we assume that the swarming crowd consists of two classes of people, one class being heavier and slower and the other class being lighter and more active, then it is obvious that the rate of progression on the street level will be more rapid for the lighter people than for the heavier people, while in the elevator the rate of progression will be identical. This gives an almost perfect analogy of what happens in the evaporative process. If we still further assume that the total numbers of the crowds ascending and descending are the same but that in the crowd ascending there is a much larger percentage of the small active people than there is in the crowd descending, we will complete the analogy. However, in the case of the evaporation from the wet bulb there is the limiting and controlling factor that the two processes, one producing an inflow of heat and the other an outflow of heat must be equal in effect, as is the case in adiabatic saturation. Since that portion of the air film actually in contact with the water surface is saturated at the temperature of the water film, and since the effect of difference in diffusion between air and water vapor is apparently small, relative to the effect of the combined diffusion and mixing process, the actual error in the assumption of equivalence of the wet-bulb and adiabatic saturation temperature is small and commercially insignificant, except possibly at high wet-bulb temperatures.

## Contact-Mixture Analogy

From the foregoing considerations there may be deduced an approximate empirical relationship based on contact and subsequent mechanical mixture. This analogy is most successfully employed in problems involving the heating and cooling of air and also to the evaporation and condensation of water vapor in connection with this process. This merits special attention because of the facility with which all problems of this character can be correlated and solved with a minimum of experimental data, to a satisfactory degree of exactitude for engineering purposes. A more detailed discussion of this valuable analogy is given in an accompanying paper by W. H. Carrier (11).

## Thermal Properties of Moist Air and Barometric Corrections

In view of the most recently accepted data on the properties of water vapor at low temperatures and the specific heat of air, it seemed desirable to prepare an exact tabulation of the psychrometric and thermal properties of moist air based on these data.

The properties of water vapor are based on the latest steam tables of Keenan and Keyes, about to be published, which are somewhat changed, at low temperature, from the older tables published by Keenan, but are more nearly in accord with the correlated values employed in Carrier's psychrometric chart 1911 (1) and with Goodenough's values.

In previous psychrometric charts by Carrier and tables by Goodenough, Swann's values for the specific heat of air were employed. More recent investigations by other methods seem to indicate that these were slightly too high. Although it is known that specific heat is not constant, the differences between the experimental values of the specific heat at constant pressure of dry air, as given by the International Critical Tables, are as great as the changes with temperature, at least within the range of temperatures important in air conditioning. Consequently a value of 0.240 is employed in the present data. This value is nearly 1 per cent lower than that previously employed. The degree of accuracy does not compare with the accuracy of the properties of water vapor, and these tables will require revision when more accurate specific-heat values are available. The specific heat of water vapor varies with the vapor pressure and the degree of superheat. For wet-bulb temperatures below 100 F and for normal degrees of superheat, it can be taken as 0.45 without causing appreciable error in psychrometric calculations. For higher wet-bulb temperatures, other values should be employed. It will be noted that the present values differ slightly from previous values, largely owing to the accepted change in specific heat of air. The present psychrometric charts can hardly be read with sufficient accuracy to distinguish the difference. For more accurate work, however, such as is required for test codes, the values given in the present tabulation should be employed.

The tabulation given in Table 1 is arranged to give the vapor pressure, the grains of moisture and the sigma function (i.e., the enthalpy of the mixture minus the enthalpy of the liquid), per pound of dry air. It also gives the grains of moisture to be subtracted from these values for each 10 F depression and, by interpolation, for any depression. As the latter relation is nearly a straight-line function, interpolation is easy and extremely exact.

This table gives all the data required for any psychrometric determination or calculation at standard barometric pressure except the correction for the enthalpy of the liquid in cooling or heating air, i.e., when there is a change in the sigma function. The latter correction is easily calculated, but for convenience Fig. 2 is given which permits reading the correction directly for any initial moisture content and for any change in wet-bulb temperature. The sigma function is, as in previous tables, a constant for any temperature of adiabatic saturation or approximately for any wet-bulb temperature. Except for the changes in constants, the same formulas are employed as in the original psychrometric charts. These are correct for temperatures of adiabatic saturation regardless of slight deviations of wet-bulb temperatures from the temperatures of adiabatic saturation.

In this paper the following symbols are employed:

$t$ = dry-bulb temperature, F

$t'$ = temperature of adiabatic saturation or corrected wet-bulb temperature, F. For convenience it will be referred to as "wet-bulb temperature"

$e$ = vapor pressure, in. Hg

$e'$ = vapor pressure corresponding to the temperature of adiabatic saturation

$P$ = barometric pressure, in. Hg

$W$ = weight of moisture per pound of dry air

$W'$ = weight of moisture per pound of dry air when saturated at a temperature corresponding to temperature of adiabatic saturation $t'$

$\Delta W$ = change in moisture content corresponding to the change in some other psychrometric quantity such as dry-bulb temperature or the barometric pressure

$\Sigma$ = the psychrometric heat function which we will term "sigma function" and which is a constant for a given wet-bulb temperature. It is the enthalpy of the mixture minus the enthalpy of the liquid

$\Delta\Sigma$ = the change in sigma function corresponding to any change in some other psychrometric factor

$\Delta H_f$ = a change in the enthalpy of the liquid per pound of dry air

$r$ = latent heat of water vapor

$C_{pa}$ = approximately 0.240 = specific heat of air at constant pressure

$C_{ps}$ = approximately 0.45 = specific heat of water vapor at constant pressure

$S$ = specific weight of water vapor

$$= \frac{\text{density of water vapor}}{\text{density of dry air at same pressure and temperature}}$$

To determine the change in moisture content $\Delta W$ corresponding to a change in wet-bulb depression, we proceed as follows:

The basic psychrometric formula is

$$(W' - W)\,r' = (C_{pa} + WC_{ps})\,(t - t') = (0.240 + 0.45\,W)\,(t - t') \quad \ldots\ [1]$$

where $W$ = lb of moisture per lb of dry air.

We may derive from Equation [1]

$$W' - W = \frac{(1680 + 0.45\,W')\,(t - t')}{r' + 0.45\,(t - t')} \quad \ldots\ [2]$$

where $W$ = *grains* of moisture per lb of dry air. These values are given in Table 1.

For an increase in barometric pressure $\Delta P$ in relation to a standard pressure, $P_0$, used in preparing the tables, and for a given value of $t'$ and $(t - t')$, the following relations may be derived

$$\Delta W' = \frac{-W'_0}{1 + \dfrac{P_0 - e'}{\Delta P}} \quad \ldots\ [3]$$

$$\Delta W = \frac{\Delta W'}{1 + \dfrac{0.45\,(t - t')}{r'}} = \frac{-W'_0}{\left[\dfrac{1}{1 + \dfrac{0.45\,(t-t')}{r'}}\right]\left[1 + \dfrac{P_0 - e'}{\Delta P}\right]} \quad \ldots\ [4]$$

or approximately

$$\Delta W = \frac{\Delta W'}{1 + \dfrac{t - t'}{2300}} \quad \ldots\ [5]$$

and

$$\Delta\Sigma = \frac{W'r'}{7000} = \frac{-W'_0\,r'}{\left(1 + \dfrac{P_0 - e'}{\Delta P}\right)7000} \quad \ldots\ [6]$$

Table 2 may be used to correct the values of $W'_0$, $W_0$, and $\Sigma_0$, found in Table 1 for a total pressure of 29.92 in. Hg abs, to any given total pressure.

The following example illustrates the use of Tables 1 and 2. Assume standard barometric conditions ($P_0$ = 29.92 in. Hg), and let it be required to find the specific humidity and the sigma function of the air at an observed dry-bulb temperature of 90 F and an observed wet-bulb temperature of 69.9 F with the air velocity over the bulb approximately 1200 fpm. The probable correction to the wet-bulb temperature is 20(0.005) = +0.1 F, so the temperature of adiabatic saturation is 69.9 + 0.1 = 70 F. From Table 1 the specific humidity $W'$ for saturation at 70 F, is 110.3 grains per lb of dry air, and $\Sigma$ = 33.40 Btu per lb of dry air. The value of $(W' - W)$ for a depression of 20 F is 32.5 grains; therefore, $W$ = 110.3 — 32.5 = 77.8 grains per lb of dry air. Now, assume the same readings were observed at a barometric pressure of 28.37 in. Hg. The corresponding barometric correction is $\Delta P$ = —1.55 in. Hg. Then, from the equations, or approximately, by interpolation from Table 2, $\Delta W'$ = + 6.2, and $W'$ = 110.3 + 6.2 = 116.5 grains per lb of dry air; $\Delta W$ = +6.1, and $W$ = 77.8 + 6.1 = 83.9 grains per lb of dry air; $\Delta\Sigma$ = 0.93, and $\Sigma$ = 33.40 + 0.93 = 34.33 Btu per lb of dry air.

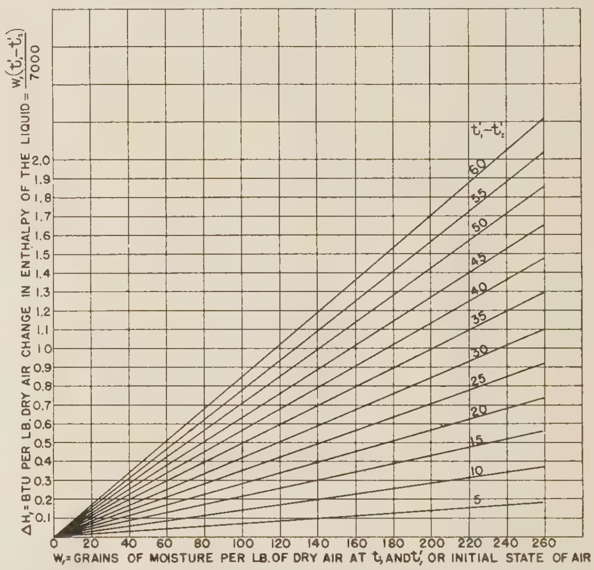


Fig. 2 Changes in Enthalpy ($\Delta H_f$) of the Liquid per Pound of Dry Air Corresponding to Changes in Temperatures of Adiabatic Saturation

(Equation: $H_1 - H_2 = (\Sigma_1 - \Sigma_2) + (W_1/7000)(t_1' - t_2') = (\Sigma_1 - \Sigma_2) + \Delta H_f$ where $H_1$ = initial enthalpy of mixture of dry air and water vapor, and $H_2$ = final enthalpy of dry air, water vapor, and condensate.)

## The Practical Value of the Σ Function

A great deal of confusion has been caused in the past in the calculation of changes in heat content of air as well as errors in the use of existing data by Carrier's unfortunate terminology, "total heat" and the labeling of the psychrometric curve of the sigma function as "total heat." Reference to the original paper (1) would have, of course, cleared up this misunderstanding. The opening reference to this in the text states:

"Total-Heat Curve. This curve shows the sensible heat in the air above a base temperature of 0 F, plus the latent heat contained in the water vapor at saturation, but not including the heat of the liquid. Since the wet-bulb temperature, or adiabatic lines contain all points having the same total heat (neglecting the heat of the liquid), the curve serves to determine the total

TABLE 1 PROPERTIES OF MOIST AIR AT STANDARD BAROMETER (29.92 IN. HG) FOR VARIOUS ADIABATIC-SATURATION (WET-BULB) TEMPERATURES $t'$ AND DRY-BULB TEMPERATURES $t$

| Temp. of adiabatic saturation $t'$ | Vapor pressure, in. Hg $e'$ | Grains per lb dry air $(t - t') = 0$ $W'$ | Sigma function $\Sigma$ | $(W' - W)$ grains moisture per lb dry air corresponding to $(t - t')$ 10 F | 20 F | 30 F | 40 F | 50 F | 60 F |
|---|---|---|---|---|---|---|---|---|---|
| 32 | 0.1803 | 26.27 | 11.73 | 15.7 | .. | .. | .. | .. | ... |
| 33 | 0.1878 | 27.5 | 12.15 | 15.7 | .. | .. | .. | .. | ... |
| 34 | 0.1955 | 28.7 | 12.56 | 15.7 | .. | .. | .. | .. | ... |
| 35 | 0.2035 | 29.89 | 12.97 | 15.7 | .. | .. | .. | .. | ... |
| 36 | 0.2118 | 31.0 | 13.39 | 15.7 | .. | .. | .. | .. | ... |
| 37 | 0.2203 | 32.3 | 13.83 | 15.7 | 31.3 | .. | .. | .. | ... |
| 38 | 0.2292 | 33.6 | 14.27 | 15.7 | 31.4 | .. | .. | .. | ... |
| 39 | 0.2383 | 34.9 | 14.70 | 15.8 | 31.4 | .. | .. | .. | ... |
| 40 | 0.2478 | 36.34 | 15.16 | 15.8 | 31.4 | .. | .. | .. | ... |
| 41 | 0.2576 | 37.8 | 15.61 | 15.8 | 31.4 | .. | .. | .. | ... |
| 42 | 0.2677 | 39.3 | 16.09 | 15.8 | 31.5 | .. | .. | .. | ... |
| 43 | 0.2782 | 40.8 | 16.55 | 15.8 | 31.5 | .. | .. | .. | ... |
| 44 | 0.2891 | 42.5 | 17.05 | 15.8 | 31.5 | .. | .. | .. | ... |
| 45 | 0.3004 | 44.13 | 17.54 | 15.9 | 31.6 | .. | .. | .. | ... |
| 46 | 0.3120 | 45.9 | 18.04 | 15.9 | 31.6 | .. | .. | .. | ... |
| 47 | 0.3240 | 47.7 | 18.55 | 15.9 | 31.6 | 47.2 | .. | .. | ... |
| 48 | 0.3364 | 49.5 | 19.06 | 15.9 | 31.6 | 47.3 | .. | .. | ... |
| 49 | 0.3493 | 51.4 | 19.59 | 15.9 | 31.7 | 47.3 | .. | .. | ... |
| 50 | 0.3626 | 53.39 | 20.13 | 15.9 | 31.7 | 47.4 | .. | .. | ... |
| 51 | 0.3674 | 55.4 | 20.67 | 15.9 | 31.7 | 47.4 | .. | .. | ... |
| 52 | 0.3906 | 57.6 | 21.23 | 16.0 | 31.8 | 47.5 | .. | .. | ... |
| 53 | 0.4052 | 59.8 | 21.81 | 16.0 | 31.8 | 47.5 | .. | .. | ... |
| 54 | 0.4203 | 62.0 | 22.38 | 16.0 | 31.8 | 47.6 | .. | .. | ... |
| 55 | 0.4359 | 64.35 | 22.97 | 16.0 | 31.9 | 47.6 | 63.3 | .. | ... |
| 56 | 0.4520 | 66.8 | 23.59 | 16.0 | 31.9 | 47.7 | 63.3 | .. | ... |
| 57 | 0.4686 | 69.3 | 24.20 | 16.0 | 32.0 | 47.8 | 63.4 | .. | ... |
| 58 | 0.4858 | 71.9 | 24.82 | 16.1 | 32.0 | 47.8 | 63.5 | .. | ... |
| 59 | 0.5035 | 74.5 | 25.46 | 16.1 | 32.0 | 47.9 | 63.6 | .. | ... |
| 60 | 0.5218 | 77.26 | 26.10 | 16.1 | 32.1 | 47.9 | 63.6 | .. | ... |
| 61 | 0.5407 | 80.1 | 26.75 | 16.1 | 32.1 | 48.0 | 63.7 | 79.3 | ... |
| 62 | 0.5601 | 83.0 | 27.43 | 16.1 | 32.2 | 48.1 | 63.8 | 79.4 | ... |
| 63 | 0.5802 | 86.1 | 28.12 | 16.2 | 32.2 | 48.1 | 63.9 | 79.5 | ... |
| 64 | 0.6009 | 89.2 | 28.82 | 16.2 | 32.3 | 48.2 | 64.0 | 79.6 | ... |
| 65 | 0.6222 | 92.47 | 29.56 | 16.2 | 32.3 | 48.3 | 64.1 | 79.8 | ... |
| 66 | 0.6442 | 95.8 | 30.27 | 16.2 | 32.4 | 48.3 | 64.1 | 79.9 | 95.4 |
| 67 | 0.6669 | 99.3 | 31.06 | 16.3 | 32.4 | 48.4 | 64.2 | 80.0 | 95.6 |
| 68 | 0.6903 | 102.8 | 31.81 | 16.3 | 32.4 | 48.5 | 64.3 | 80.1 | 95.7 |
| 69 | 0.7144 | 106.5 | 32.61 | 16.3 | 32.5 | 48.5 | 64.4 | 80.2 | 95.9 |
| 70 | 0.7392 | 110.3 | 33.40 | 16.3 | 32.5 | 48.6 | 64.5 | 80.3 | 96.0 |
| 71 | 0.7648 | 114.3 | 34.24 | 16.4 | 32.6 | 48.7 | 64.6 | 80.4 | 96.1 |
| 72 | 0.7912 | 118.3 | 35.10 | 16.4 | 32.6 | 48.8 | 64.7 | 80.6 | 96.3 |
| 73 | 0.8183 | 122.4 | 35.94 | 16.4 | 32.7 | 48.8 | 64.8 | 80.7 | 96.4 |
| 74 | 0.8462 | 126.7 | 36.80 | 16.4 | 32.7 | 48.9 | 64.9 | 80.8 | 96.6 |
| 75 | 0.8750 | 131.2 | 37.71 | 16.5 | 32.8 | 49.0 | 65.0 | 81.0 | 96.7 |
| 76 | 0.9046 | 135.8 | 38.64 | 16.5 | 32.9 | 49.1 | 65.2 | 81.1 | 96.9 |
| 77 | 0.9352 | 140.5 | 39.57 | 16.5 | 32.9 | 49.2 | 65.3 | 81.2 | 97.1 |
| 78 | 0.9666 | 145.4 | 40.53 | 16.6 | 33.0 | 49.3 | 65.4 | 81.4 | 97.2 |
| 79 | 0.9989 | 150.4 | 41.50 | 16.6 | 33.0 | 49.3 | 65.5 | 81.5 | 97.4 |
| 80 | 1.0321 | 155.6 | 42.51 | 16.6 | 33.1 | 49.4 | 65.6 | 81.7 | 97.6 |
| 81 | 1.0664 | 160.9 | 43.53 | 16.7 | 33.2 | 49.5 | 65.8 | 81.8 | 97.8 |
| 82 | 1.1016 | 166.4 | 44.57 | 16.7 | 33.2 | 49.6 | 65.9 | 82.0 | 98.0 |
| 83 | 1.1378 | 172.1 | 45.67 | 16.7 | 33.3 | 49.7 | 66.0 | 82.2 | 98.2 |
| 84 | 1.1750 | 178.0 | 46.77 | 16.8 | 33.4 | 49.8 | 66.2 | 82.3 | 98.4 |
| 85 | 1.2133 | 184.1 | 47.91 | 16.8 | 33.4 | 49.9 | 66.3 | 82.5 | 98.6 |
| 86 | 1.2527 | 190.4 | 49.07 | 16.8 | 33.5 | 50.0 | 66.4 | 82.7 | 98.8 |
| 87 | 1.2931 | 196.9 | 50.26 | 16.9 | 33.6 | 50.1 | 66.6 | 82.9 | 99.0 |
| 88 | 1.3347 | 203.5 | 51.48 | 16.9 | 33.6 | 50.2 | 66.7 | 83.1 | 99.3 |
| 89 | 1.3775 | 210.3 | 52.72 | 17.0 | 33.7 | 50.4 | 66.9 | 83.2 | 99.5 |
| 90 | 1.4215 | 217.4 | 53.99 | 17.0 | 33.8 | 50.5 | 67.0 | 83.4 | 99.7 |
| 91 | 1.4667 | 224.7 | 55.30 | 17.0 | 33.9 | 50.7 | 67.2 | 83.6 | 99.9 |
| 92 | 1.5131 | 232.2 | 56.63 | 17.1 | 34.0 | 50.8 | 67.3 | 83.8 | 100.2 |
| 93 | 1.5608 | 239.9 | 57.99 | 17.1 | 34.0 | 50.9 | 67.5 | 84.1 | 100.4 |
| 94 | 1.6097 | 247.9 | 59.39 | 17.1 | 34.1 | 51.0 | 67.7 | 84.3 | 100.6 |
| 95 | 1.6600 | 256.0 | 60.84 | 17.2 | 34.2 | 51.2 | 67.9 | 84.5 | 100.9 |
| 96 | 1.7117 | 264.5 | 62.31 | 17.2 | 34.3 | 51.3 | 68.0 | 84.7 | 101.2 |
| 97 | 1.7647 | 273.2 | 63.82 | 17.3 | 34.4 | 51.4 | 68.2 | 84.9 | 101.5 |
| 98 | 1.8192 | 282.2 | 65.37 | 17.3 | 34.5 | 51.5 | 68.4 | 85.2 | 101.8 |
| 99 | 1.8751 | 291.5 | 66.96 | 17.4 | 34.6 | 51.6 | 68.6 | 85.4 | 102.1 |
| 100 | 1.9325 | 301.0 | 68.61 | 17.4 | 34.7 | 51.7 | 68.8 | 85.7 | 102.4 |

TABLE 2 ADDITIVE CORRECTIONS TO STANDARD TABULAR VALUES OF $W'$, $W$, AND $\Sigma$ FOR DEVIATIONS IN BAROMETRIC PRESSURE FROM 29.92 IN. HG

| $t'$, F | $\Delta P = +1$ $\Delta W'$ | $\Delta\Sigma$ | $\Delta P = -1$ $\Delta W'$ | $\Delta\Sigma$ | $\Delta P = -2$ $\Delta W'$ | $\Delta\Sigma$ | $\Delta P = -3$ $\Delta W'$ | $\Delta\Sigma$ | $\Delta P = -4$ $\Delta W'$ | $\Delta\Sigma$ | $\Delta P = -5$ $\Delta W'$ | $\Delta\Sigma$ | $\Delta P = -6$ $\Delta W'$ | $\Delta\Sigma$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 32 | −0.9 | −0.14 | 0.9 | 0.14 | 1.9 | 0.29 | 3.0 | 0.45 | 4.1 | 0.63 | 5.3 | 0.82 | 6.7 | 1.03 |
| 33 | −0.9 | −0.14 | 1.0 | 0.15 | 2.0 | 0.31 | 3.1 | 0.47 | 4.3 | 0.66 | 5.6 | 0.86 | 7.0 | 1.07 |
| 34 | −0.9 | −0.14 | 1.0 | 0.15 | 2.1 | 0.32 | 3.2 | 0.49 | 4.5 | 0.68 | 5.8 | 0.89 | 7.3 | 1.12 |
| 35 | −1.0 | −0.15 | 1.0 | 0.16 | 2.2 | 0.33 | 3.4 | 0.52 | 4.7 | 0.71 | 6.1 | 0.93 | 7.6 | 1.16 |
| 36 | −1.0 | −0.15 | 1.1 | 0.17 | 2.3 | 0.35 | 3.5 | 0.54 | 4.8 | 0.74 | 6.3 | 0.97 | 7.9 | 1.21 |
| 37 | −1.0 | −0.16 | 1.1 | 0.17 | 2.4 | 0.37 | 3.6 | 0.56 | 5.0 | 0.77 | 6.6 | 1.01 | 8.2 | 1.26 |
| 38 | −1.1 | −0.17 | 1.2 | 0.18 | 2.4 | 0.38 | 3.8 | 0.58 | 5.2 | 0.80 | 6.8 | 1.05 | 8.5 | 1.31 |
| 39 | −1.1 | −0.17 | 1.2 | 0.19 | 2.5 | 0.39 | 3.9 | 0.61 | 5.4 | 0.83 | 7.1 | 1.09 | 8.9 | 1.36 |
| 40 | −1.2 | −0.18 | 1.3 | 0.20 | 2.6 | 0.40 | 4.1 | 0.63 | 5.7 | 0.87 | 7.4 | 1.13 | 9.2 | 1.41 |
| 41 | −1.2 | −0.19 | 1.3 | 0.20 | 2.7 | 0.41 | 4.3 | 0.65 | 5.9 | 0.90 | 7.7 | 1.18 | 9.6 | 1.47 |
| 42 | −1.3 | −0.20 | 1.4 | 0.21 | 2.8 | 0.43 | 4.4 | 0.67 | 6.1 | 0.93 | 8.0 | 1.23 | 10.0 | 1.53 |
| 43 | −1.3 | −0.20 | 1.4 | 0.22 | 2.9 | 0.45 | 4.6 | 0.70 | 6.4 | 0.97 | 8.4 | 1.28 | 10.4 | 1.59 |
| 44 | −1.4 | −0.21 | 1.5 | 0.23 | 3.1 | 0.47 | 4.8 | 0.73 | 6.6 | 1.01 | 8.7 | 1.33 | 10.8 | 1.65 |
| 45 | −1.4 | −0.22 | 1.5 | 0.23 | 3.2 | 0.49 | 5.0 | 0.76 | 6.9 | 1.05 | 9.0 | 1.38 | 11.2 | 1.71 |
| 46 | −1.5 | −0.23 | 1.6 | 0.24 | 3.3 | 0.50 | 5.2 | 0.79 | 7.2 | 1.19 | 9.4 | 1.43 | 11.7 | 1.78 |
| 47 | −1.6 | −0.24 | 1.7 | 0.25 | 3.4 | 0.52 | 5.4 | 0.82 | 7.5 | 1.13 | 9.7 | 1.48 | 12.1 | 1.85 |
| 48 | −1.6 | −0.25 | 1.7 | 0.26 | 3.6 | 0.54 | 5.6 | 0.85 | 7.7 | 1.17 | 10.1 | 1.54 | 12.6 | 1.92 |
| 49 | −1.7 | −0.26 | 1.8 | 0.27 | 3.7 | 0.56 | 5.8 | 0.88 | 8.0 | 1.22 | 10.4 | 1.59 | 13.1 | 1.99 |
| 50 | −1.8 | −0.27 | 1.9 | 0.28 | 3.9 | 0.59 | 6.0 | 0.92 | 8.3 | 1.27 | 10.9 | 1.65 | 13.6 | 2.06 |
| 51 | −1.8 | −0.28 | 1.9 | 0.29 | 4.0 | 0.61 | 6.3 | 0.96 | 8.1 | 1.32 | 11.3 | 1.71 | 14.1 | 2.14 |
| 52 | −1.9 | −0.29 | 2.0 | 0.30 | 4.2 | 0.64 | 6.5 | 0.99 | 9.0 | 1.37 | 11.7 | 1.17 | 14.6 | 2.22 |
| 53 | −2.0 | −0.30 | 2.1 | 0.32 | 4.3 | 0.66 | 6.7 | 1.03 | 9.4 | 1.43 | 12.2 | 1.85 | 15.2 | 2.31 |
| 54 | −2.0 | −0.31 | 2.2 | 0.33 | 4.5 | 0.68 | 7.0 | 1.07 | 9.7 | 1.48 | 12.7 | 1.92 | 15.8 | 2.40 |
| 55 | −2.1 | −0.32 | 2.3 | 0.34 | 4.7 | 0.71 | 7.3 | 1.11 | 10.1 | 1.53 | 13.1 | 1.99 | 16.4 | 2.50 |
| 56 | −2.2 | −0.33 | 2.3 | 0.35 | 4.8 | 0.73 | 7.6 | 1.15 | 10.5 | 1.59 | 13.7 | 2.07 | 17.1 | 2.59 |
| 57 | −2.3 | −0.35 | 2.4 | 0.36 | 5.0 | 0.76 | 7.9 | 1.20 | 10.9 | 1.65 | 14.2 | 2.15 | 17.8 | 2.69 |
| 58 | −2.4 | −0.36 | 2.5 | 0.38 | 5.2 | 0.79 | 8.2 | 1.24 | 11.3 | 1.71 | 14.8 | 2.23 | 18.4 | 2.78 |
| 59 | −2.5 | −0.37 | 2.6 | 0.39 | 5.4 | 0.82 | 8.5 | 1.28 | 11.7 | 1.77 | 15.3 | 2.31 | 19.1 | 2.88 |
| 60 | −2.5 | −0.38 | 2.7 | 0.41 | 5.6 | 0.85 | 8.8 | 1.33 | 12.2 | 1.84 | 15.8 | 2.39 | 19.8 | 2.99 |
| 61 | −2.6 | −0.39 | 2.8 | 0.42 | 5.9 | 0.89 | 9.1 | 1.38 | 12.6 | 1.91 | 16.4 | 2.48 | 20.6 | 3.10 |
| 62 | −2.7 | −0.41 | 2.9 | 0.44 | 6.1 | 0.92 | 9.5 | 1.43 | 13.1 | 1.98 | 17.0 | 2.57 | 21.3 | 3.22 |
| 63 | −2.8 | −0.42 | 3.0 | 0.46 | 6.3 | 0.95 | 9.8 | 1.48 | 13.6 | 2.05 | 17.7 | 2.67 | 22.1 | 3.35 |
| 64 | −2.9 | −0.44 | 3.2 | 0.48 | 6.5 | 0.98 | 10.2 | 1.53 | 14.1 | 2.13 | 18.3 | 2.77 | 23.0 | 3.48 |
| 65 | −3.1 | −0.46 | 3.3 | 0.50 | 6.8 | 1.02 | 10.6 | 1.59 | 14.6 | 2.21 | 19.0 | 2.87 | 23.8 | 3.60 |
| 66 | −3.2 | −0.48 | 3.4 | 0.51 | 7.0 | 1.06 | 11.0 | 1.65 | 15.2 | 2.29 | 19.7 | 2.98 | 24.7 | 3.73 |
| 67 | −3.3 | −0.50 | 3.5 | 0.53 | 7.3 | 1.10 | 11.4 | 1.72 | 15.8 | 2.38 | 20.5 | 3.09 | 25.6 | 3.86 |
| 68 | −3.4 | −0.51 | 3.7 | 0.55 | 7.5 | 1.14 | 11.8 | 1.78 | 16.3 | 2.46 | 21.2 | 3.20 | 26.6 | 4.01 |
| 69 | −3.5 | −0.53 | 3.8 | 0.57 | 7.8 | 1.18 | 12.3 | 1.85 | 16.9 | 2.55 | 22.0 | 3.32 | 27.6 | 4.15 |
| 70 | −3.7 | −0.55 | 3.9 | 0.59 | 8.1 | 1.22 | 12.7 | 1.91 | 17.5 | 2.64 | 22.8 | 3.44 | 28.6 | 4.29 |
| 71 | −3.8 | −0.57 | 4.1 | 0.61 | 8.4 | 1.26 | 13.1 | 1.98 | 18.2 | 2.74 | 23.7 | 3.56 | 29.6 | 4.45 |
| 72 | −3.9 | −0.59 | 4.2 | 0.63 | 8.7 | 1.31 | 13.6 | 2.05 | 18.9 | 2.84 | 24.5 | 3.69 | 30.7 | 4.62 |
| 73 | −4.1 | −0.61 | 4.4 | 0.66 | 9.1 | 1.36 | 14.1 | 2.12 | 19.6 | 2.95 | 25.4 | 3.83 | 31.8 | 4.79 |
| 74 | −4.2 | −0.63 | 4.5 | 0.68 | 9.4 | 1.41 | 14.6 | 2.19 | 20.3 | 3.05 | 26.4 | 3.96 | 33.0 | 4.96 |
| 75 | −4.4 | −0.66 | 4.7 | 0.71 | 9.7 | 1.46 | 15.1 | 2.27 | 21.0 | 3.15 | 27.3 | 4.10 | 34.2 | 5.13 |
| 76 | −4.5 | −0.68 | 4.9 | 0.73 | 10.1 | 1.51 | 15.7 | 2.35 | 21.7 | 3.25 | 28.3 | 4.24 | 35.4 | 5.31 |
| 77 | −4.7 | −0.70 | 5.0 | 0.76 | 10.4 | 1.56 | 16.2 | 2.43 | 22.4 | 3.36 | 29.3 | 4.39 | 36.7 | 5.50 |
| 78 | −4.9 | −0.73 | 5.2 | 0.78 | 10.8 | 1.62 | 16.8 | 2.52 | 23.2 | 3.48 | 30.4 | 4.55 | 38.0 | 5.70 |
| 79 | −5.0 | −0.75 | 5.4 | 0.81 | 11.2 | 1.67 | 17.4 | 2.61 | 24.1 | 3.61 | 31.5 | 4.71 | 39.4 | 5.90 |
| 80 | −5.2 | −0.78 | 5.6 | 0.84 | 11.6 | 1.73 | 18.0 | 2.70 | 25.0 | 3.74 | 32.6 | 4.87 | 40.8 | 6.11 |
| 81 | −5.4 | −0.81 | 5.8 | 0.87 | 12.0 | 1.79 | 18.7 | 2.79 | 25.9 | 3.88 | 33.7 | 5.04 | 42.3 | 6.32 |
| 82 | −5.6 | −0.84 | 6.0 | 0.90 | 12.5 | 1.86 | 19.3 | 2.89 | 26.8 | 4.01 | 34.9 | 5.22 | 43.8 | 6.54 |
| 83 | −5.8 | −0.87 | 6.2 | 0.93 | 12.9 | 1.92 | 20.0 | 2.99 | 27.8 | 4.15 | 36.2 | 5.41 | 45.3 | 6.77 |
| 84 | −6.0 | −0.90 | 6.4 | 0.96 | 13.3 | 1.99 | 20.7 | 3.10 | 28.8 | 4.29 | 37.5 | 5.60 | 47.0 | 7.01 |
| 85 | −6.2 | −0.93 | 6.7 | 1.00 | 13.8 | 2.06 | 21.5 | 3.21 | 29.8 | 4.44 | 38.9 | 5.80 | 48.6 | 7.25 |
| 86 | −6.4 | −0.96 | 6.9 | 1.03 | 14.2 | 2.13 | 22.3 | 3.32 | 30.9 | 4.60 | 40.2 | 6.00 | 50.4 | 7.51 |
| 87 | −6.7 | −1.00 | 7.1 | 1.06 | 14.7 | 2.20 | 23.0 | 3.43 | 32.0 | 4.76 | 41.7 | 6.21 | 52.2 | 7.78 |
| 88 | −6.9 | −1.03 | 7.4 | 1.10 | 15.3 | 2.28 | 23.9 | 3.55 | 33.1 | 4.93 | 43.1 | 6.43 | 54.1 | 8.06 |
| 89 | −7.1 | −1.06 | 7.6 | 1.13 | 15.8 | 2.36 | 24.7 | 3.68 | 34.3 | 5.10 | 44.7 | 6.66 | 56.0 | 8.35 |
| 90 | −7.4 | −1.10 | 7.9 | 1.17 | 16.4 | 2.44 | 25.6 | 3.81 | 35.5 | 5.28 | 46.3 | 6.89 | 58.0 | 8.64 |
| 91 | −7.6 | −1.13 | 8.2 | 1.21 | 11.0 | 2.53 | 26.5 | 3.94 | 36.8 | 5.47 | 47.9 | 7.13 | 60.0 | 8.94 |
| 92 | −7.9 | −1.17 | 8.5 | 1.26 | 17.6 | 2.62 | 27.4 | 4.08 | 38.1 | 5.66 | 49.6 | 7.38 | 62.2 | 9.25 |
| 93 | −8.2 | −1.22 | 8.8 | 1.31 | 18.2 | 2.71 | 28.4 | 4.22 | 39.4 | 5.86 | 51.4 | 7.64 | 64.4 | 9.57 |
| 94 | −8.5 | −1.27 | 9.1 | 1.35 | 18.8 | 2.80 | 29.4 | 4.37 | 40.8 | 6.07 | 53.2 | 7.91 | 66.6 | 9.90 |
| 95 | −8.8 | −1.31 | 9.4 | 1.40 | 19.5 | 2.90 | 30.4 | 4.52 | 42.2 | 6.28 | 55.0 | 8.19 | 69.0 | 10.25 |
| 96 | −9.1 | −1.35 | 9.8 | 1.45 | 20.2 | 3.00 | 31.5 | 4.68 | 43.7 | 6.50 | 57.0 | 8.48 | 71.5 | 10.62 |
| 97 | −9.4 | −1.40 | 10.1 | 1.50 | 21.0 | 3.11 | 32.6 | 4.85 | 45.3 | 6.73 | 59.0 | 8.77 | 74.0 | 10.99 |
| 98 | −9.7 | −1.44 | 10.4 | 1.55 | 21.7 | 3.21 | 33.7 | 5.02 | 46.9 | 6.96 | 61.1 | 9.07 | 76.6 | 11.36 |
| 99 | −10.1 | −1.49 | 10.8 | 1.60 | 22.4 | 3.32 | 34.9 | 5.19 | 48.5 | 7.20 | 63.2 | 9.38 | 79.3 | 11.76 |
| 100 | −10.4 | −1.54 | 11.1 | 1.65 | 23.2 | 3.43 | 36.1 | 5.37 | 50.2 | 7.44 | 65.5 | 9.70 | 82.1 | 12.18 |

NOTE. $\Delta W$ is about 1 per cent less than $\Delta W'$ for $t - t' = 23$ F and proportionally less for other values of $(t - t')$.

heat in the air under any and all conditions represented by the chart." This term "total heat," should, of course, not have been used, even with the explanation because it already had a very definite meaning as employed in tables of the properties of vapors and did include the heat of the liquid.

In that text, the example following this statement, however, is defective since it does not include the change in the heat of the liquid. However, the necessity of including this in exact calculations was appreciated. This was brought out in the closure when referring to the discussion by Gunn (6), wherein it is stated:

"Mr. Gunn emphasizes the necessity of taking into account the heat of the liquid when dehumidifying air by means of a cold-water spray or in heating. This has not been taken into account in the total-heat curve for a very good reason. It always has to be calculated independently, because invariably the condensed moisture will be carried through a different range from the air. The heat of the liquid, however, is but a small percentage of the total heat so that only a small percentage correction is necessary."

This, perhaps is not an adequate explanation for this omission. In the first place, a line of constant temperature of adiabatic saturation on the psychrometric chart is not a line of constant enthalpy. A property of the mixture that does remain exactly constant for a constant temperature of adiabatic saturation is that property designated by the symbol Σ in the author's 1911 paper (1), and defined as

$$\Sigma = 0.24t' + \frac{W'r'}{7000} = 0.24t + \frac{W}{7000}\left[r' + 0.45\,(t - t')\right]$$

where $\Sigma$ = Btu per lb of dry air; $t$ = dry-bulb temperature, F; $t'$ = temperature of adiabatic saturation, F; $r'$ = latent heat of the vapor at $t'$, Btu per lb; $W'$ = specific humidity for saturation at $t'$, grains per lb of dry air; and $W$ = actual specific humidity, grains per lb of dry air.

In other words, the Σ function is equal to the true enthalpy of the mixture minus the enthalpy of the liquid at the temperature of adiabatic saturation. The lines of constant Σ on the psychrometric chart connect all those states of the mixture for which the temperature of adiabatic saturation is the same. They also represent the adiabatic process of humidification with the liquid supplied at the temperature of adiabatic saturation. The observed wet-bulb temperature may be corrected, if necessary, by the amount indicated elsewhere in this paper, to give the temperature of adiabatic saturation. Then, with the temperature of adiabatic saturation and the total pressure known, the Σ function may readily be obtained.

In calculating the heat supplied during heating processes or the heat removed during cooling processes, the Σ function may be used provided a correction is made for the noninclusion of the enthalpy of the liquid. For example, the heat added per pound of dry air in a heating and humidifying process is

$$\begin{aligned} Q_h &= h_2 - h_1 - \left(\frac{W_2 - W_1}{7000}\right) h_{f3} \\ &= \Sigma_2 - \Sigma_1 + \frac{W_2}{7000}\,(h_{f2} - h_{f3}) - \frac{W_1}{7000}\,(h_{f1} - h_{f3}) \\ &= \Sigma_2 - \Sigma_1 + \frac{W_2}{7000}\,(t'_1 - t_3) - \frac{W_1}{7000}\,(t'_1 - t_3) \end{aligned}$$

where $t_3$ is the temperature of the water supplied for humidifying, and the primed temperatures are those of adiabatic saturation (wet-bulb temperature).

For cooling and dehumidifying process, the quantity of heat removed per pound of dry air is

$$\begin{aligned} Q_c &= h_1 - h_2 - \left(\frac{W_1 - W_2}{7000}\right) h_{f3} \\ &= \Sigma_1 - \Sigma_2 + \frac{W_1}{7000}\,(t'_1 - t_3) - \frac{W_2}{7000}\,(t'_2 - t_3) \end{aligned}$$

where $t_3$ is the temperature at which the condensate is removed from the surface.

For precise calculations based upon the use of psychrometric tables, the principal objection to the use of the true enthalpy of the mixture is that this enthalpy changes with dry-bulb temperatures at constant temperature of adiabatic saturation (or wet bulb). Therefore, the tables must include values for the enthalpy of the mixture at all degrees of partial saturation. Illustrations of the use of the Σ function as contrasted with the use of enthalpy follow:

In all of the examples which follow, the wet-bulb temperature will be assumed equal to the temperature of adiabatic saturation.

*Example No. 1.* Air at a dry bulb of 95 F, a wet bulb of 75 F, and a total pressure of 29.92 in. Hg abs is cooled to a saturated state at 45 F; the condensate is removed at 45 F. Required: the quantity of heat removed in Btu per lb of dry air.

(*a*) Using the Σ function: $\Sigma_1 = 37.71$; $W_1 = 131.2 - 32.8 = 98.4$; $\Sigma_2 = 17.54$; and therefore

$$Q_c = 37.71 - 17.54 + \frac{98.4}{7000}\,(75 - 45)$$

$= 20.17 + 0.42 = 20.59$ Btu per lb of dry air. Fig. 2 has been prepared to assist in making this calculation. In this chart, $W_1\,(t_1' - t_2')$ is plotted against $W_1$ for different values of $t_1' - t_2'$. For the example cited, enter the chart at $W_1 = 98.4$, go vertically to $t_1' - t_2' = 30$, and read 0.42, which is the amount to be added to the difference in the Σ function to get the heat removed. The tables and this chart eliminate slide-rule work from this calculation and substitute the simplest arithmetic, addition and subtraction.

(*b*) Using enthalpy: The enthalpy of a partially saturated mixture is not precisely equal to the enthalpy of a mixture saturated at the same wet bulb. The variation of enthalpy with dry bulb at a constant wet bulb of 75 F for standard barometer is

| $t'$ | $t$ | $t - t'$ | $H$ | $H' - H$ | $\Sigma$ |
|---|---|---|---|---|---|
| 75 | 75 | 0 | 38.52 | 0.00 | 37.71 |
| 75 | 85 | 10 | 38.41 | 0.11 | 37.71 |
| 75 | 95 | 20 | 38.31 | 0.21 | 37.71 |
| 75 | 105 | 30 | 38.21 | 0.31 | 37.71 |
| 75 | 115 | 40 | 38.12 | 0.40 | 37.71 |

so that $H_1 = 38.31$; $W_1 = 98.4$; $H_2 = 17.62$; $W_2 = 44.1$; and therefore

$$Q_c = 38.31 - 17.62 - \frac{(98.4 - 44.1)}{7000}\,(13) = 20.69 - 0.10 = 20.59$$

The same result is secured, of course, with either method of correct calculation. The advantage of the Σ function is that the extra tabular values for conditions of partial saturation are eliminated. Such enthalpy tables are not practical, and any engineer who wishes to use the enthalpy method might better calculate the enthalpy from the fundamental relation, $H = 0.24t + Wh_g/7000$. The extra work of correcting the Σ differences due to noninclusion of the enthalpy of the liquid just about balances the extra work required to correct the enthalpy differences for the enthalpy of the condensate.

*Example No. 2.* Saturated air at 80 F and standard barometric pressure is cooled to a saturated state at 60 F; the condensate is removed at 60 F. Required: the quantity of heat removed in Btu per lb of dry air.

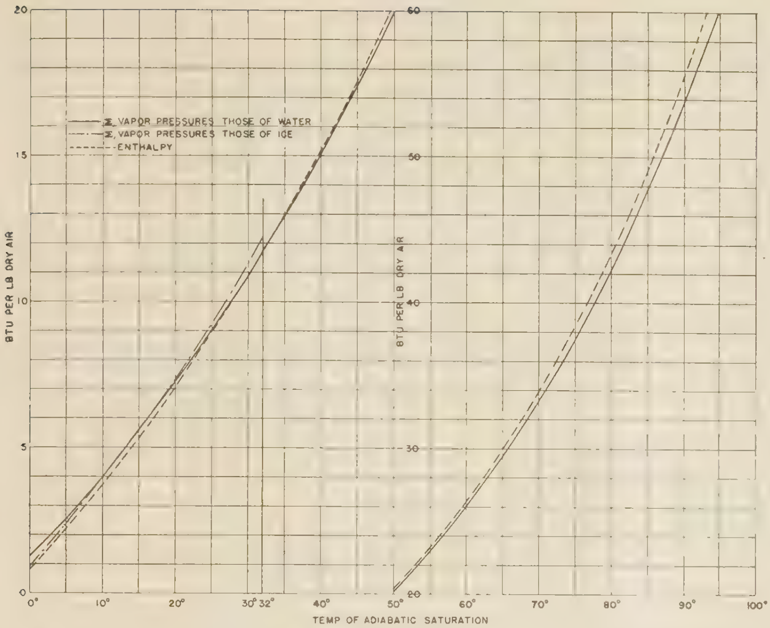


FIG. 3 COMPARISON OF THE SIGMA FUNCTION AND ENTHALPY FOR SATURATED AIR-VAPOR MIXTURES

(*a*) Using the Σ function: $\Sigma_1 = 42.51$; $\Sigma_2 = 26.10$; $W_1 \dfrac{(t_1' - t_2')}{7000} = 0.45$ (from chart); and therefore

$$Q_c = 42.51 - 26.10 + 0.45 = 16.86$$

(*b*) Using enthalpies: $H_1 = 43.58$ (calculated from formula); $H_2 = 26.41$; $W_1 = 155.6$; $W_2 = 77.3$; enthalpy of the condensate $= \dfrac{155.6 - 77.3}{7000}(60 - 32)$; and therefore

$$Q_c = 43.58 - 26.41 - \frac{(155.6 - 77.3)}{7000}(60 - 32)$$
$$= 17.17 - 0.31 = 16.86$$

*Example No. 3.* Air at a dry bulb of 95 F, a wet bulb of 65 F, and a total pressure of 29.92 in. Hg abs is cooled to a saturated state at 45 F; the condensate is removed at 45 F. Required: the quantity of heat removed in Btu per pound of dry air.

(*a*) Using Σ function: $\Sigma_1 = 29.56$; $\Sigma_2 = 17.54$; $W_1 \dfrac{(t_1' - t_2')}{7000} = 0.13$ (from chart); and therefore

$$Q_c = 29.56 - 17.54 + 0.13 = 12.15$$

(*b*) Using the enthalpies: $H_1 = 29.77$; $H_2 = 17.62$; $W_1 = 44.2$; $W_2 = 44.1$; and therefore

$$Q_c = 29.77 - 17.62 - \frac{(44.2 - 44.1)}{7000}(13) = 12.15$$

Some will be interested in the liberties that may be taken with these exact solutions. For example, if the Σ function be used, what error is introduced by ignoring the correction for non-inclusion of the enthalpy of the liquid? If the enthalpy tables be used, what error is introduced by the double neglect of the variation of enthalpy at constant wet bulb and also of the enthalpy of the condensate? These errors are indicated as follows:

| Example no. | Correct value for heat removed | $\Sigma_1 - \Sigma_2$ | $H_1' - H_2$ |
|---|---|---|---|
| 1 | 20.59 | 20.17 | 20.90 |
| 2 | 16.86 | 16.41 | 17.17 |
| 3 | 12.15 | 12.02 | 12.38 |

### THE Σ FUNCTION BELOW THE FREEZING POINT

There has been considerable controversy regarding the so-called "total heat" curve when extended below 32 F. This has risen from a misunderstanding of the definition or interpretation of total heat as formerly used. This total-heat curve, as explicitly stated by Carrier, excluded the enthalpy of the liquid. For this important psychrometric property we now propose the term "the sigma function" or "sigma" to distinguish it from total-heat content or enthalpy. This is defined in the original paper (1) as the sensible heat in the air above base temperature of 0 F plus the latent heat contained in the water vapor at saturation. Others have assumed that the so-called total-heat curve did or should contain the heat of the liquid. This, as we have shown is not desirable in psychrometric calculations. Fig. 3 gives the correct curve for the sigma function of air for temperatures of adiabatic saturation between 0 and 100 F. This, as will be noted, is a discontinuous curve at 32 F. Above 32 F it is given for the evaporation over water and below 32 F, for evaporation over ice. The enthalpy curve for the mixture is, however, a continuous curve. The sigma curve for values above 32 F can be extrapolated as a continuous curve for temperatures below 32 F which represent evaporation from subcooled water. Both of these conditions can actually be secured in practice although this can best be illustrated by observing the action of a wet bulb in an air current in a room in which the wet-bulb temperature is below 32 F, as for example in an egg cold-storage room.

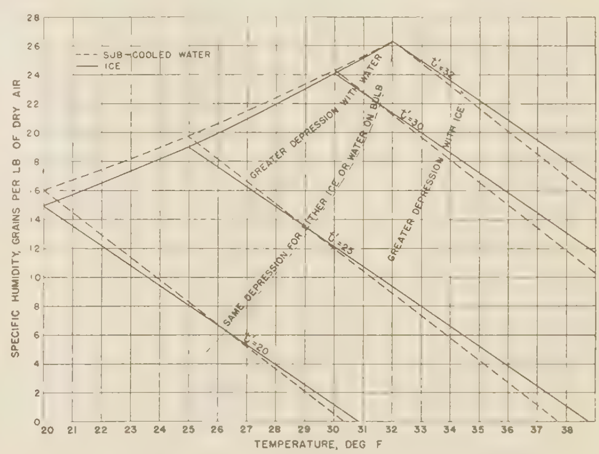


FIG. 4 COMPARISON OF ADIABATIC-SATURATION (WET-BULB) TEMPERATURES OVER ICE WITH THOSE OVER WATER

After dipping the wet bulb in water slightly above 32 F and exposing it to air motion, the reading will drop rapidly and continuously to a minimum temperature below 32 F. This minimum temperature occurs without freezing the water on the bulb. Suddenly after reaching this minimum temperature, the reading will jump abruptly back to 32 F where it will remain for some length of time, until the water on the wick has completely frozen, then it will gradully fall to a second minimum where it will remain permanently until the ice on the bulb is evaporated. The second wet-bulb reading, where evaporation occurs over ice,

will be either higher than the previous reading, or lower, or the same, depending upon the degree of humidity. For example, in an egg cold-storage room with a wet-bulb temperature of 28 F and relative humidity between 85 and 90 per cent, the second wet-bulb reading will be higher than the first, while at 50 per cent relative humidity, for example, and the same wet-bulb temperature, the second reading will be lower than the first. At 32 F, the wet-bulb reading over ice is always lower than the wet-bulb reading over water. These variations of wet-bulb depressions whether the wick is still wet or frozen, are due, first, to the change of sigma value which is greater for ice than for water, and second, the opposing effect of lower vapor pressures over ice below 32 F as compared with those of subcooled water. The variations in wet-bulb depressions over water and over ice in the region below 32 F are illustrated graphically in Fig. 4.

It is interesting to note that below 32 F, air may be actually saturated at two different temperatures; the first being the saturation condition over ice and the second, the saturation condition over subcooled water. If air is cooled and humidified at the same time, it is possible to approach saturation at a vapor pressure above that over ice. The air is clear and will not fog but it is not a stable condition as a contact with any object in the room will produce condensation or frosting. Air saturated over ice will produce a wet-bulb depression with a sling pyschrometer if the reading is taken before the ice freezes on the bulb. These are all very interesting and instructive phenomena in this region below freezing.

The argument has been offered that the enthalpy of saturated or partially saturated air forms a continuous curve and not a discontinuous curve. This statement is as correct for enthalpy of moist air as it is for enthalpy of water vapor, but the sigma function does not form a continuous curve. The reason for this is that the abrupt change in sigma is exactly counterbalanced by a corresponding abrupt negative change in the enthalpy of the liquid. The real cause for the abrupt change in the latent heat of the vapor at 32 F is the sudden change in slope in the vapor-pressure curve that occurs at this point, as shown in Fig. 4. Referring to this figure, it will be seen that there is necessarily quite a different slope ($dp/dT$) for the two pressure curves at this point, but the vapor pressures are identical; the specific weights of the paper are identical and the specific volume of the vapor is identical, therefore, in the Clapeyron equation, which must of necessity be thermodynamically valid, we find the latent heats to be in direct proportion to the slope. If these slopes are correctly calculated, the difference in their ratio will correspond to 144 Btu per lb, or exactly the latent heat of ice.

A study of the interesting psychrometric phenomena in the region below 32 F emphasizes again the psychrometric significance of the sigma function. It is certain that wet-bulb readings, below 32 F have approximately the same relationship to the temperature of adiabatic saturation as they do above 32 F. Since the error is a function of the depression and the depression below 32 F is necessarily small, an error in the assumption that the wet-bulb temperature over ice is the same as the temperature of adiabatic saturation over ice, is a negligible error in comparison with the accuracy of observation. Therefore, the authors believe that the calculated values of adiabatic saturation can be used with confidence as equivalent to wet-bulb temperature for psychrometric readings in this region. A tabulation of the thermal and psychrometric properties at standard barometric pressures of air below 32 F is given in Table 4.

Table 5 gives the corrections in specific humidity and sigma below 32 F for differences in barometric pressure.

## Conclusions

1 The temperature of adiabatic saturation is a most important criterion as well as a most useful function in psychrometric tables and engineering calculations.

2 The observed wet-bulb reading at higher velocities are below the temperature of adiabatic saturation rather than above as previously assumed. The credit for this determination and

TABLE 3 PROPERTIES OF MOIST AIR AT STANDARD BAR. (29.92 IN. HG) FOR VARIOUS ADIABATIC-SATURATION (WET-BULB) TEMPERATURES $t'$ AND DRY-BULB TEMPERATURES $t$

(Vapor pressures are those of subcooled water)

| | | | | $W' - W$ corresponding to $t - t'$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $t'$, F | $e'$ | $W'$ | Σ | 2 F | 4 F | 6 F | 8 F | 10 F | 12 F | 14 F | 16 F |
| 10 | 0.0706 | 10.3 | 4.00 | 3.1 | 6.2 | 9.3 | .. | .. | .. | .. | ... |
| 11 | 0.0739 | 10.8 | 4.31 | 3.1 | 6.2 | 9.3 | .. | . | | | |
| 12 | 0.0773 | 11.3 | 4.63 | 3.1 | 6.2 | 9.3 | .. | | | | |
| 13 | 0.0809 | 11.8 | 4.95 | 3.1 | 6.2 | 9.3 | .. | | | | |
| 14 | 0.0846 | 12.3 | 5.27 | 3.1 | 6.2 | 9.3 | .. | | | | . |
| 15 | 0.0884 | 12.9 | 5.60 | 3.1 | 6.2 | 9.3 | 12.4 | | | | .. |
| 16 | 0.0923 | 13.5 | 5.93 | 3.1 | 6.2 | 9.3 | 12.4 | | | | |
| 17 | 0.0964 | 14.1 | 6.26 | 3.1 | 6.2 | 9.3 | 12.4 | | | | . |
| 18 | 0.1006 | 14.7 | 6.60 | 3.1 | 6.2 | 9.3 | 12.4 | . | | | . |
| 19 | 0.1050 | 15.3 | 6.94 | 3.1 | 6.2 | 9.3 | 12.4 | .. | | | |
| 20 | 0.1096 | 16.0 | 7.28 | 3.1 | 6.2 | 9.3 | 12.4 | 15.5 | .. | | . |
| 21 | 0.1144 | 16.7 | 7.62 | 3.1 | 6.2 | 9.3 | 12.4 | 15.5 | .. | | . |
| 22 | 0.1194 | 17.4 | 7.97 | 3.1 | 6.2 | 9.3 | 12.4 | 15.5 | .. | | . |
| 23 | 0.1245 | 18.2 | 8.33 | 3.1 | 6.2 | 9.3 | 12.5 | 15.6 | .. | .. | . |
| 24 | 0.1298 | 19.0 | 8.69 | 3.1 | 6.2 | 9.4 | 12.5 | 15.6 | 18.7 | .. | . |
| 25 | 0.1353 | 19.8 | 9.05 | 3.1 | 6.3 | 9.4 | 12.5 | 15.6 | 18.7 | .. | .. |
| 26 | 0.1410 | 20.6 | 9.41 | 3.1 | 6.3 | 9.4 | 12.5 | 15.6 | 18.7 | .. | .. |
| 27 | 0.1470 | 21.5 | 9.79 | 3.1 | 6.3 | 9.4 | 12.5 | 15.6 | 18.7 | .. | . |
| 28 | 0.1532 | 22.4 | 10.17 | 3.1 | 6.3 | 9.4 | 12.5 | 15.6 | 18.7 | 21.8 | .. |
| 29 | 0.1597 | 23.3 | 10.55 | 3.1 | 6.3 | 9.4 | 12.5 | 15.6 | 18.7 | 21.8 | ... |
| 30 | 0.1663 | 24.3 | 10.94 | 3.1 | 6.3 | 9.4 | 12.5 | 15.6 | 18.8 | 21.9 | ... |
| 31 | 0.1732 | 25.3 | 11.33 | 3.1 | 6.3 | 9.4 | 12.5 | 15.6 | 18.8 | 21.9 | 25.00 |
| 32 | 0.1803 | 26.4 | 11.73 | 3.1 | 6.3 | 9.4 | 12.5 | 15.7 | 18.8 | 21.9 | 25.00 |

TABLE 4 PROPERTIES OF MOIST AIR AT STANDARD BAR. (29.92 IN. HG) FOR VARIOUS ADIABATIC-SATURATION (WET-BULB) TEMPERATURES $t'$ AND DRY-BULB TEMPERATURES $t$

(Vapor pressures are those of ice)

| | | | | $W' - W$ corresponding to $t - t'$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $t'$, F | $e'$ | $W'$ | Σ | 1 F | 2 F | 4 F | 6 F | 8 F | 10 F | 12 F | 14 F |
| −40 | 0.0039 | 0.55 | −9.50 | .. | .. | ... | ... | .. | .. | .. | .. |
| −35 | 0.0052 | 0.75 | −8.27 | .. | .. | ... | ... | | .. | . | .. |
| −30 | 0.0070 | 1.02 | −7.02 | .. | .. | | . | | | | .. |
| −25 | 0.0094 | 1.37 | −5.76 | .. | .. | | . | | | . | .. |
| −20 | 0.0126 | 1.84 | −4.48 | 1.38 | .. | . | .. | | | | .. |
| −15 | 0.0167 | 2.44 | −3.17 | 1.38 | 2.75 | .. | ... | . | | . | . |
| −10 | 0.0220 | 3.22 | −1.84 | 1.38 | 2.75 | ... | ... | | . | .. | .. |
| − 5 | 0.0289 | 4.21 | −0.47 | 1.38 | 2.75 | ... | .. | . | | .. | .. |
| 0 | 0.0377 | 5.49 | +0.96 | 1.4 | 2.8 | ... | .. | . | | | . |
| 1 | 0.0397 | 5.8 | 1.25 | 1.4 | 2.8 | 5.5 | .. | | | | . |
| 2 | 0.0419 | 6.1 | 1.55 | 1.4 | 2.8 | 5.5 | ... | | | | . |
| 3 | 0.0441 | 6.4 | 1.84 | 1.4 | 2.8 | 5.5 | . | | | | . |
| 4 | 0.0464 | 6.7 | 2.14 | 1.4 | 2.8 | 5.5 | .. | | | .. | .. |
| 5 | 0.0488 | 7.1 | 2.44 | 1.4 | 2.8 | 5.5 | ... | | | | .. |
| 6 | 0.0514 | 7.5 | 2.74 | 1.4 | 2.8 | 5.5 | ... | | | . | .. |
| 7 | 0.0542 | 7.9 | 3.15 | 1.4 | 2.8 | 5.5 | ... | | | .. | . |
| 8 | 0.0570 | 8.3 | 3.46 | 1.4 | 2.8 | 5.5 | 8.3 | | | | .. |
| 9 | 0.0599 | 8.7 | 3.78 | 1.4 | 2.8 | 5.5 | 8.3 | | | . | |
| 10 | 0.0629 | 9.1 | 3.99 | 1.4 | 2.8 | 5.5 | 8.3 | .. | .. | . | . |
| 11 | 0.0661 | 9.6 | 4.31 | 1.4 | 2.8 | 5.5 | 8.3 | .. | .. | | . |
| 12 | 0.0695 | 10.1 | 4.64 | 1.4 | 2.8 | 5.5 | 8.3 | .. | .. | | .. |
| 13 | 0.0730 | 10.6 | 4.97 | 1.4 | 2.8 | 5.5 | 8.3 | .. | .. | | . |
| 14 | 0.0767 | 11.2 | 5.31 | 1.4 | 2.8 | 5.5 | 8.3 | 11.0 | .. | | .. |
| 15 | 0.0806 | 11.8 | 5.65 | 1.4 | 2.8 | 5.5 | 8.3 | 11.0 | .. | . | . |
| 16 | 0.0847 | 12.4 | 5.99 | 1.4 | 2.8 | 5.5 | 8.3 | 11.0 | .. | | . |
| 17 | 0.0889 | 13.1 | 6.34 | 1.4 | 2.8 | 5.5 | 8.3 | 11.0 | .. | | . |
| 18 | 0.0933 | 13.7 | 6.69 | 1.4 | 2.8 | 5.5 | 8.3 | 11.0 | .. | | .. |
| 19 | 0.0979 | 14.4 | 7.05 | 1.4 | 2.8 | 5.5 | 8.3 | 11.0 | 13.8 | | |
| 20 | 0.1028 | 15.0 | 7.41 | 1.4 | 2.8 | 5.5 | 8.3 | 11.0 | 13.8 | .. | .. |
| 21 | 0.1078 | 15.7 | 7.78 | 1.4 | 2.8 | 5.5 | 8.3 | 11.0 | 13.8 | .. | .. |
| 22 | 0.1131 | 16.5 | 8.16 | 1.4 | 2.8 | 5.5 | 8.3 | 11.0 | 13.8 | .. | .. |
| 23 | 0.1186 | 17.3 | 8.54 | 1.4 | 2.8 | 5.5 | 8.3 | 11.0 | 13.8 | 16.6 | .. |
| 24 | 0.1243 | 18.2 | 8.92 | 1.4 | 2.8 | 5.5 | 8.3 | 11.0 | 13.8 | 16.6 | .. |
| 25 | 0.1303 | 19.0 | 9.32 | 1.4 | 2.8 | 5.5 | 8.3 | 11.0 | 13.8 | 16.6 | .. |
| 26 | 0.1366 | 19.9 | 9.71 | 1.4 | 2.8 | 5.5 | 8.3 | 11.0 | 13.8 | 16.6 | 19.3 |
| 27 | 0.1432 | 20.9 | 10.17 | 1.4 | 2.8 | 5.5 | 8.3 | 11.0 | 13.8 | 16.6 | 19.3 |
| 28 | 0.1500 | 21.9 | 10.57 | 1.4 | 2.8 | 5.5 | 8.3 | 11.1 | 13.8 | 16.6 | 19.3 |
| 29 | 0.1571 | 23.0 | 10.98 | 1.4 | 2.8 | 5.5 | 8.3 | 11.1 | 13.8 | 16.6 | 19.3 |
| 30 | 0.1645 | 24.1 | 11.39 | 1.4 | 2.8 | 5.5 | 8.3 | 11.1 | 13.8 | 16.7 | 19.3 |
| 31 | 0.1723 | 25.2 | 11.83 | 1.4 | 2.8 | 5.5 | 8.3 | 11.1 | 13.8 | 16.7 | 19.3 |
| 32 | 0.1803 | 26.4 | 12.27 | 1.4 | 2.8 | 5.5 | 8.3 | 11.1 | 13.8 | 16.7 | 19.3 |

TABLE 5 ADDITIVE CORRECTIONS TO STANDARD TABULAR VALUES OF $W'$, $W$, AND $\Sigma$ FOR DEVIATIONS IN BAROMETRIC PRESSURE FROM 29.92 IN. HG

(Vapor pressures are those of ice)

| | $\Delta P = +1$ | | $\Delta P = -1$ | | $\Delta P = -2$ | | $\Delta P = -3$ | | $\Delta P = -4$ | | $\Delta P = -5$ | | $\Delta P = -6$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $t'$, F | $\Delta W$ | $\Delta\Sigma$ | $\Delta W$ | $\Delta\Sigma$ | $\Delta W$ | $\Delta\Sigma$ | $\Delta W$ | $\Delta\Sigma$ | $\Delta W$ | $\Delta\Sigma$ | $\Delta W$ | $\Delta\Sigma$ | $\Delta W$ | $\Delta\Sigma$ |
| −40 | −0.00 | 0.00 | 0.01 | 0.00 | 0.04 | 0.01 | 0.07 | 0.01 | 0.09 | 0.02 | 0.11 | 0.02 | 0.14 | 0.02 |
| −35 | −0.01 | 0.00 | 0.02 | 0.00 | 0.05 | 0.01 | 0.08 | 0.01 | 0.12 | 0.02 | 0.15 | 0.03 | 0.19 | 0.03 |
| −30 | −0.03 | −0.01 | 0.03 | −0.01 | 0.08 | 0.01 | 0.11 | 0.02 | 0.16 | 0.03 | 0.20 | 0.04 | 0.26 | 0.05 |
| −25 | −0.04 | −0.01 | 0.04 | −0.01 | 0.11 | 0.02 | 0.17 | 0.03 | 0.21 | 0.04 | 0.27 | 0.05 | 0.35 | 0.06 |
| −20 | −0.06 | −0.01 | 0.06 | −0.01 | 0.15 | 0.02 | 0.23 | 0.04 | 0.28 | 0.05 | 0.37 | 0.06 | 0.46 | 0.08 |
| −15 | −0.08 | −0.02 | 0.08 | −0.01 | 0.20 | 0.03 | 0.31 | 0.05 | 0.38 | 0.07 | 0.49 | 0.08 | 0.61 | 0.11 |
| −10 | −0.11 | −0.02 | 0.11 | −0.02 | 0.24 | 0.04 | 0.40 | 0.06 | 0.50 | 0.09 | 0.65 | 0.11 | 0.81 | 0.14 |
| − 5 | −0.14 | −0.02 | 0.14 | −0.02 | 0.31 | 0.05 | 0.50 | 0.08 | 0.65 | 0.11 | 0.85 | 0.15 | 1.06 | 0.18 |
| 0 | −0.17 | −0.03 | 0.17 | 0.03 | 0.37 | 0.07 | 0.60 | 0.11 | 0.84 | 0.15 | 1.10 | 0.19 | 1.38 | 0.22 |
| 1 | −0.2 | −0.03 | 0.2 | 0.04 | 0.4 | 0.07 | 0.6 | 0.11 | 0.9 | 0.16 | 1.1 | 0.20 | 1.4 | 0.23 |
| 2 | −0.2 | −0.03 | 0.2 | 0.04 | 0.4 | 0.08 | 0.7 | 0.12 | 0.9 | 0.17 | 1.2 | 0.21 | 1.4 | 0.25 |
| 3 | −0.2 | −0.03 | 0.2 | 0.04 | 0.5 | 0.08 | 0.7 | 0.13 | 1.0 | 0.17 | 1.2 | 0.22 | 1.5 | 0.26 |
| 4 | −0.2 | −0.04 | 0.2 | 0.04 | 0.5 | 0.09 | 0.8 | 0.13 | 1.0 | 0.18 | 1.3 | 0.23 | 1.6 | 0.28 |
| 5 | −0.2 | −0.04 | 0.2 | 0.04 | 0.5 | 0.09 | 0.8 | 0.14 | 1.1 | 0.19 | 1.4 | 0.24 | 1.7 | 0.29 |
| 6 | −0.2 | −0.04 | 0.3 | 0.05 | 0.5 | 0.09 | 0.8 | 0.15 | 1.2 | 0.20 | 1.4 | 0.25 | 1.8 | 0.31 |
| 7 | −0.3 | −0.04 | 0.3 | 0.05 | 0.6 | 0.10 | 0.9 | 0.15 | 1.2 | 0.21 | 1.5 | 0.26 | 1.9 | 0.32 |
| 8 | −0.3 | −0.05 | 0.3 | 0.05 | 0.6 | 0.10 | 0.9 | 0.16 | 1.3 | 0.23 | 1.6 | 0.28 | 2.0 | 0.34 |
| 9 | −0.3 | −0.05 | 0.3 | 0.05 | 0.6 | 0.11 | 1.0 | 0.17 | 1.4 | 0.24 | 1.7 | 0.29 | 2.1 | 0.37 |
| 10 | −0.3 | −0.05 | 0.3 | 0.05 | 0.7 | 0.11 | 1.0 | 0.18 | 1.4 | 0.25 | 1.8 | 0.30 | 2.2 | 8.39 |
| 11 | −0.3 | −0.05 | 0.3 | 0.05 | 0.7 | 0.12 | 1.1 | 0.19 | 1.5 | 0.26 | 1.9 | 0.32 | 2.4 | 0.41 |
| 12 | −0.3 | −0.06 | 0.4 | 0.06 | 0.7 | 0.13 | 1.1 | 0.20 | 1.6 | 0.27 | 2.0 | 0.34 | 2.5 | 0.4[illegible] |
| 13 | −0.3 | −0.06 | 0.4 | 0.06 | 0.8 | 0.13 | 1.2 | 0.21 | 1.7 | 0.29 | 2.1 | 0.36 | 2.7 | 0.46 |
| 14 | −0.4 | −0.06 | 0.4 | 0.07 | 0.8 | 0.14 | 1.2 | 0.22 | 1.7 | 0.30 | 2.2 | 0.38 | 2.8 | 0.49 |
| 15 | −0.4 | −0.07 | 0.4 | 0.07 | 0.8 | 0.15 | 1.3 | 0.23 | 1.8 | 0.32 | 2.3 | 0.40 | 3.0 | 0.52 |
| 16 | −0.4 | −0.07 | 0.4 | 0.07 | 0.9 | 0.15 | 1.4 | 0.24 | 1.9 | 0.33 | 2.5 | 0.43 | 3.1 | 0.54 |
| 17 | −0.4 | −0.07 | 0.4 | 0.08 | 0.9 | 0.16 | 1.5 | 0.26 | 2.0 | 0.35 | 2.7 | 0.45 | 3.3 | 0.57 |
| 18 | −0.4 | −0.08 | 0.5 | 0.08 | 1.0 | 0.17 | 1.5 | 0.27 | 2.1 | 0.36 | 2.8 | 0.48 | 3.5 | 0.61 |
| 19 | −0.5 | −0.08 | 0.5 | 0.09 | 1.0 | 0.18 | 1.6 | 0.28 | 2.2 | 0.38 | 3.0 | 0.51 | 3.7 | 0.64 |
| 20 | −0.5 | −0.08 | 0.5 | 0.09 | 1.1 | 0.19 | 1.7 | 0.29 | 2.3 | 0.40 | 3.1 | 0.53 | 3.8 | 0.67 |
| 21 | −0.5 | −0.09 | 0.5 | 0.10 | 1.1 | 0.20 | 1.8 | 0.30 | 2.4 | 0.42 | 3.2 | 0.56 | 4.0 | .70 |
| 22 | −0.5 | −0.09 | 0.6 | 0.10 | 1.2 | 0.21 | 1.8 | 0.32 | 2.6 | 0.44 | 3.4 | 0.59 | 4.2 | 0.73 |
| 23 | −0.6 | −0.10 | 0.6 | 0.10 | 1.2 | 0.22 | 1.9 | 0.34 | 2.7 | 0.47 | 3.5 | 0.62 | 4.5 | 0.7 |
| 24 | −0.6 | −0.10 | 0.6 | 0.11 | 1.3 | 0.23 | 2.0 | 0.35 | 2.8 | 0.49 | 3.7 | 0.65 | 4.7 | 0.81 |
| 25 | −0.6 | −0.11 | 0.7 | 0.11 | 1.4 | 0.24 | 2.1 | 0.37 | 3.0 | 0.52 | 3.9 | 0.68 | 4.9 | 0.85 |
| 26 | −0.7 | −0.11 | 0.7 | 0.12 | 1.4 | 0.25 | 2.2 | 0.39 | 3.1 | 0.54 | 4.1 | 0.71 | 5.2 | 0.90 |
| 27 | −0.7 | −0.12 | 0.7 | 0.13 | 1.5 | 0.26 | 2.3 | 0.41 | 3.2 | 0.57 | 4.3 | 0.75 | 5.4 | 0.94 |
| 28 | −0.7 | −0.13 | 0.8 | 0.13 | 1.6 | 0.27 | 2.5 | 0.43 | 3.4 | 0.59 | 4.5 | 0.78 | 5.6 | 0.98 |
| 29 | −0.8 | −0.13 | 0.8 | 0.14 | 1.7 | 0.29 | 2.6 | 0.45 | 3.6 | 0.62 | 4.7 | 0.82 | 5.9 | 1.03 |
| 30 | −0.8 | −0.14 | 0.8 | 0.15 | 1.7 | 0.30 | 2.7 | 0.47 | 3.7 | 0.65 | 4.9 | 0.85 | 6.1 | 1.07 |
| 31 | −0.8 | −0.14 | 0.9 | 0.16 | 1.8 | 0.32 | 2.8 | 0.49 | 3.9 | 0.68 | 5.1 | 0.89 | 6.4 | 1.12 |
| 32 | −0.9 | −0.15 | 0.9 | 0.18 | 1.9 | 0.33 | 3.0 | 0.52 | 4.1 | 0.70 | 5.3 | 0.92 | 6.7 | 1 17 |

NOTE: These corrections are approximately true when vapor pressures are those of subcooled water.

the rationalization of the underlying theory belongs to Arnold. His contribution not only clarifies the phenomena of the wet-bulb temperature and correlates it with the temperature of adiabatic saturation, but also is most valuable in predicting and interpreting the results obtained in heat transmission with moist air.

3 While the observed wet-bulb temperatures, under certain conditions, may be lower than the temperature of adiabatic saturation, the present accepted deviation is less than previously assumed. This is fully confirmed by the data of both Arnold and Dropkin, with water evaporating into air. The best method of employing psychrometric charts or tables, is to make a correction of the wet-bulb temperature corresponding to the velocity to obtain the true temperature of adiabatic saturation and then to determine the psychrometric values from the latter temperature. This is due to the fact that the correction varies somewhat with the velocity while the temperature of adiabatic saturation is a constant. For extremely accurate results, it is important to measure the velocity at which the wet-bulb temperature reading is taken.

4 The difference between the enthalpy of the mixture and that of the liquid, i.e., the sigma function, is more useful and practicable in thermal calculations than the enthalpy alone. Enthalpy cannot be employed without complete tabulation of enthalpies for all percentages of saturation or use of an enthalpy chart. In addition, a correction has to be made for the enthalpy of the condensate. By employing the sigma function, a single table or curve may be employed for the saturated vapor which will apply to all wet-bulb temperatures and needs only to be corrected for the change in enthalpy of the liquid to give the true difference in enthalpy.

5 Below the freezing point, air may have two states of saturation—that over water and that over ice. Two different wet-bulb temperatures may be observed for the same moisture content, depending upon whether the water on the bulb remains liquid or is frozen. In this the wet-bulb reading follows the laws of adiabatic saturation.

6 The empirical contact-mixture theory can be applied most advantageously in the solution of all problems of heating, cooling, evaporation and condensation with a mixture of air and water vapor under forced convection.

## Appendix 1

### THE ARNOLD THEORY

When a wet-bulb thermometer is placed in a stream of gas which is in turbulent motion, there is a layer of gas near the thermometer in laminar or viscous motion. At greater distances from the thermometer, the type of motion changes from laminar to turbulent, and beyond this the motion is entirely turbulent. The convection theory applied to the wet-bulb hygrometer by August assumes that a film of saturated gas surrounds the wet bulb and that the transfer of heat and vapor occurs by convection; the resistance to these processes in the laminar film is neglected. The diffusion theory proposed by Maxwell assumes that the transfer of heat and vapor is limited by the speed of the molecular processes of conduction and diffusion and that resistance to transfer of heat and vapor external to the laminar film is negligible. Each of these theories represents a limiting condition; the August theory probably holds very well at high velocities of the gas, while the Maxwell theory is adequate at zero gas velocity, only, however, if there is no transfer of heat by radiation to the liquid on the bulb. Neither theory completely describes

the phenomena that actually exist at common velocities, such as those involved when a sling psychrometer is used.

Dr. Arnold (2) has applied the Prandtl analogy between heat transfer and friction and developed a more complete theory that combines the resistances, both to transfer of heat and vapor, of the laminar and turbulent zones. According to this theory, the coefficient of heat transfer is given by

$$h_g = \frac{k\left(\dfrac{C\mu}{k}\right)}{B_H\left(\dfrac{C\mu}{k} + \dfrac{1-r}{r}\right)} \qquad [1]$$

where $h_g$ = the coefficient of heat transfer; $k$ = the specific thermal conductivity of the gas-vapor mixture; $\frac{C\mu}{k}$ = the Prandtl number for the gas; $B_H$ = the equivalent thickness of an assumed laminar layer in resisting flow of heat; and $r$ = the ratio of the fluid velocity at the boundary between the laminar and turbulent layers to the average velocity of the main fluid stream.

A similar theory applied to the transfer of vapor gives the following expression for the coefficient of vapor transfer

$$f_g = \frac{\left(\dfrac{D\rho}{M_g}\right)\left(\dfrac{\mu}{D\rho}\right)}{B_D\left(\dfrac{\mu}{D\rho} + \dfrac{1-r}{r}\right)} \qquad [2]$$

where $f_g$ = the coefficient of vapor transfer; $D$ = the coefficient of vapor diffusion, $\rho$ the density of the gas; $\mu$ = the absolute viscosity of the gas; $M_g$ = the molecular weight of the gas; and $B_D$ = the equivalent thickness of an assumed laminar layer in resisting the transfer of vapor.

Many of the equations written for the wet-bulb hygrometer are of the following form

$$\frac{e' - e}{P(t - t')} = A \qquad [3]$$

where $e'$ = the saturation pressure of the vapor at the wet-bulb temperature; $e$ = the partial pressure of the vapor in the mixture; $P$ = the total pressure of the mixture; $t$ = the dry-bulb temperature; and $t'$ = the wet-bulb temperature. By experimental observation of these five factors, $A$ may be found.

However, the Arnold theory may be completed to predict the value of $A$. The liquid on the wet bulb attains a steady temperature as a consequence of the equilibrium that exists between the rate of heat transfer from the gas to the liquid and the rate at which heat is utilized in evaporating that liquid. If radiation is ignored, then

$$A' = \frac{h_g}{f_g\, M_v\, r'} \qquad [4]$$

where $M_v$ = the molecular weight of the liquid, and $r'$ = the latent heat of vaporization of the liquid at the wet-bulb temperature.

A complete equation is obtained by combining Equations [1], [2], and [4], so that

$$A' = \frac{B_D\, M_a C\left(\dfrac{\mu}{D\rho} + \dfrac{1-r}{r}\right)}{B_H\, M_v r'\left(\dfrac{C\mu}{k} + \dfrac{1-r}{r}\right)} \qquad [5]$$

If there is any radiation to the wet bulb, $A$ is greater than $A'$, and this effect can be included in the theory, since

$$A = A'\left(1 + \frac{h_r}{h_g}\right) \qquad [6]$$

where $h_r$ = the coefficient of heat transfer by radiation, and $h_g$ = the coefficient of heat transfer by convection as given in Equation [1].

The value of $A$ predicted by this theory is

$$A = \frac{B_D M_a C\left(\dfrac{\mu}{D\rho} + \dfrac{1-r}{r}\right)}{B_H M_v r'\left(\dfrac{C\mu}{k} + \dfrac{1-r}{r}\right)}\left(1 + \frac{h_r}{h_g}\right) \qquad [7]$$

Dr. Arnold assumed that $B_H = B_D$, in other words, that the equivalent thickness of the laminar layer in resisting the transfer of heat is the same as the equivalent thickness of this hypothetical laminar layer resisting the transfer of vapor, and his final equation for the wet-bulb hygrometer is

$$\frac{e' - e}{P(t - t')} = \frac{M_a C\left(\dfrac{\mu}{D\rho} + \dfrac{1-r}{r}\right)}{M_v r'\left(\dfrac{C\mu}{k} + \dfrac{1-r}{r}\right)}\left(1 + \frac{h_r}{h_g}\right) \qquad [8]$$

### The Process of Adiabatic Saturation

In the process of adiabatic saturation of a gas-vapor mixture a liquid is supplied to a saturator, and the mixture of gas and vapor becomes saturated during the flow through the saturator with no external transfer of heat. The principle of conservation of energy, when applied to such a process, will predict the temperature of adiabatic saturation $t_2$. If there is no appreciable change in elevation or velocity of the fluids, no external supply or removal of heat or mechanical energy, and if the liquid is supplied at the temperature of adiabatic saturation, the energy balance has been shown by Carrier (1) and others to give the following equation

$$\frac{\dfrac{e_2}{P - e_2} - \dfrac{e}{P - e}}{t - t_2} = \frac{M_g C}{M_v r_2} \qquad [9]$$

where $e_2$ = the saturation pressure of the vapor at $t_2$
$e$ = the partial pressure of the vapor in the original mixture
$P$ = the total pressure of the mixture
$t$ = the initial dry-bulb temperature
$t_2$ = the temperature of adiabatic saturation
$M_g$ = the molecular weight of the gas
$M_v$ = the molecular weight of the vapor
$C$ = the humid specific heat of the mixture
$r_2$ = the latent heat of vaporization of the liquid at $t_2$

For values of $e$ and $e_2$ small in comparison with $P$, this equation may also be written, in approximate form, as

$$\frac{e_2 - e}{P(t - t_2)} = \frac{M_g C}{M_v r_2} \qquad [10]$$

Since $e_2$ and $r_2$ depend only upon $t_2$, the temperature of adiabatic saturation is given for any mixture of known initial condition. This temperature is fixed, regardless of the relative rates of transfer of heat and vapor, as long as the mixture is adiabatically saturated. The energy balance, alone, will not fix the terminal state of the mixture for incomplete saturation, however.

*The Wet-Bulb Temperature and the Temperature of Adiabatic Saturation.* A comparison of Equations [3], [7], and [10] shows that the wet-bulb temperature $t'$ will equal the temperature of adiabatic saturation when

$$\frac{B_D}{B_H}\left(\frac{\frac{\mu}{D}+\frac{1-r}{r}}{\frac{C\mu}{k}+\frac{1-r}{r}}\right)\left(1+\frac{h_r}{h_g}\right)=1.0 \qquad [11]$$

If we follow the Arnold theory to the last step and assume $B_D = B_H$, then $t' = t_2$ when

$$\left(\frac{\frac{\mu}{D\rho}+\frac{1-r}{r}}{\frac{C\mu}{k}+\frac{1-r}{r}}\right)\left(1+\frac{h_r}{h_g}\right)=1.0 \qquad [12]$$

To simplify the discussion that follows, let

$$E=\frac{\frac{\mu}{D\rho}+\frac{1-r}{r}}{\frac{c\mu}{k}+\frac{1-r}{r}},$$

and

$$F=1+\frac{h_r}{h_g}$$

For a given liquid-gas system the value of $E$ is determined principally by the velocity of the gas past the wet bulb. At zero velocity, the value of $\frac{1-r}{r}$ is zero, and $E$ becomes equal to $\frac{k}{C\rho D}$, or the ratio of thermal diffusivity to vapor diffusivity, which is the result predicted by the Maxwell theory. For a given system $r$ is inversely proportional (7), approximately, to the one-eighth power of the velocity and at infinite velocities $\frac{1-r}{r}$ becomes infinitely large and $E = 1$, which is the result predicted by the August theory. At all intermediate velocities, the resistance of the laminar film to the transfer of heat and vapor decreases more rapidly than the resistance offered by the turbulent zone when the velocity increases. The value of the correction for the radiation effect $F$ depends primarily upon the wet-bulb temperature and the gas velocity. At constant velocity, $F$ increases as the wet-bulb temperature increases; at constant wet-bulb temperature, $F$ decreases as the velocity increases and approaches unity at very high velocities. Therefore, for any gas-vapor system having values of $E$ less than one, since $F$ is unity or higher there may be some combination of velocity and wet-bulb temperature giving a product $EF$ equal to one. In other words, there may be some one velocity at a given wet-bulb temperature where the wet-bulb temperature and the temperature of adiabatic saturation will be identical.

*The Wet-Bulb Hygrometer for Water-Air Mixtures.* The various properties of the air and water vapor, necessary in applying the previous general theory to this particular mixture, change slightly with changes in temperature, but for air-conditioning calculations it will be sufficiently accurate to take these properties at one temperature, say 60 F. The following values will be used in the calculations:

$\rho$ = 0.075 lb per cu ft = 0.00233 slugs per cu ft
$C$ = 0.242 Btu per lb per deg F = 7.8 Btu per slug per deg F
$\mu$ = 3.74 (10 — 7) lb sec per sq ft
$k$ = 0.014 Btu ft per hr per sq ft per deg F
= 3.9 $(10)^{-6}$ Btu ft per sec per sq ft per deg F
$D$ = 0.00026 sq ft per sec
$\frac{k}{C\rho}$ = 0.000215 sq ft per sec
$\frac{C\mu}{k}$ = 0.748
$\frac{\mu}{D\rho}$ = 0.616.

The assumed diameter of the thermometer is $d$ = 0.02 ft, and Reynolds' number will then be

$$\frac{dV\rho}{\mu}=2.08\,V$$

where $V$ = the air velocity, fpm.

The coefficient of heat transfer by radiation for the case of a wet-bulb thermometer in a relatively large enclosure has been shown (8) to be $h_r = 4\,(p)\,(0.00172)\left(\frac{T'}{100}\right)^3$ where $h_r$ = the coefficient of heat transfer by radiation, Btu per hr per sq ft per deg F; $p$ = the emissivity of the surface; and $T'$ = the wet-bulb temperature, F abs.

Using $p$ = 0.9 for the water-covered surface

$$h_r = 0.0062\left(\frac{T'}{100}\right)^3$$

Values of $h_r$ are given in Table A1 for several different wet-bulb temperatures.

TABLE A1 THE COEFFICIENT OF HEAT TRANSFER BY RADIATION

| Wet-bulb temperature, F | $h_r$ Btu per hr per sq ft per deg F |
|---|---|
| 40 | 0.775 |
| 50 | 0.825 |
| 60 | 0.875 |
| 70 | 0.925 |
| 80 | 0.980 |

Ulsamer (9) and McAdams (10) have correlated the results of King, Hughes, Reiher, and others for the coefficient of heat transfer for the flow of air normal to single cylinders. Between values of Reynolds' number of 10 and 40,000, these results may be very closely fitted by the following empirical equation

$$\frac{h_g d}{k}=0.48\left(\frac{dV\rho}{\mu}\right)^{0.52}$$

For the assumed properties of air and the thermometer diameter, this equation becomes

$$h_g = 0.49V^{0.52}$$

Values of $h_g$ are given in Table A2 for several different velocities.

TABLE A2 COEFFICIENT OF HEAT TRANSFER BY CONVECTION FROM AIR TO THERMOMETER

| Air velocity $V$, fpm | Coefficient of heat transfer $h_g$, Btu per hr per sq ft per deg F |
|---|---|
| 10000 | 59.0 |
| 3000 | 31.0 |
| 2000 | 25.0 |
| 1000 | 17.7 |
| 500 | 12.5 |
| 400 | 11.0 |
| 300 | 9.5 |
| 200 | 7.7 |
| 100 | 5.4 |
| 20 | 2.3 |

Dr. Arnold determined the value of the psychrometric coefficient $A$ by experiments with toluene, chlorobenzene, and xylene as the liquids, and then calculated $E$ for each liquid at several different velocities. Since the Prandtl number for air and the velocity distribution as indicated by $r$ are not materially affected by the liquid on the wet bulb, a plot of $E_1$ for one liquid versus $E_2$ for another liquid should be a straight line passing through the points (1, 1) and $\left(\frac{k}{C\rho D_2}, \frac{k}{C\rho D_1}\right)$. The experi-

mental date for chlorobenzene and xylene verified this theory and also gave a velocity scale for the plot, from which it is possible to determine the value of $E$ for water at different velocities. Such values are given in Table A3.

TABLE A3 VALUES OF $E$ FOR WATER

| Air velocity $V$, fpm | $E$ |
|---|---|
| 10000 | 0.947 |
| 3000 | 0.930 |
| 2000 | 0.923 |
| 1000 | 0.913 |
| 500 | 0.905 |
| 400 | 0.902 |
| 300 | 0.898 |
| 200 | 0.892 |
| 100 | 0.884 |
| 20 | 0.852 |

TABLE A4 RESULTS OF ARNOLD THEORY

| Wet-bulb temp, $t'$, F | Radiation coefficient, Btu per hr per sq ft $h_r$ | Air velocity, fpm $V$ | Convection coefficient, Btu per hr per sq ft per deg F $h_a$ | $F$ | $E$ | $EF$ |
|---|---|---|---|---|---|---|
| 40 | 0.775 | 10000 | 59.0 | 1.0131 | 0.947 | 0.959 |
| | | 3000 | 31.0 | 1.0250 | 0.930 | 0.953 |
| | | 2000 | 25.0 | 1.0310 | 0.923 | 0.952 |
| | | 1000 | 17.7 | 1.0438 | 0.913 | 0.953 |
| | | 500 | 12.5 | 1.0620 | 0.905 | 0.961 |
| | | 400 | 11.0 | 1.0705 | 0.902 | 0.966 |
| | | 300 | 9.5 | 1.0816 | 0.898 | 0.971 |
| | | 200 | 7.7 | 1.1006 | 0.892 | 0.982 |
| | | 100 | 5.4 | 1.1435 | 0.884 | 1.011 |
| | | 20 | 2.3 | 1.3370 | 0.852 | 1.139 |
| 50 | 0.825 | 10000 | 59.0 | 1.0140 | 0.947 | 0.960 |
| | | 3000 | 31.0 | 1.0266 | 0.930 | 0.955 |
| | | 2000 | 25.0 | 1.0330 | 0.923 | 0.953 |
| | | 1000 | 17.7 | 1.0466 | 0.913 | 0.956 |
| | | 500 | 12.5 | 1.0660 | 0.905 | 0.965 |
| | | 400 | 11.0 | 1.0750 | 0.902 | 0.970 |
| | | 300 | 9.5 | 1.0868 | 0.898 | 0.976 |
| | | 200 | 7.7 | 1.1071 | 0.892 | 0.988 |
| | | 100 | 5.4 | 1.1528 | 0.884 | 1.019 |
| | | 20 | 2.3 | 1.3587 | 0.852 | 1.158 |
| 60 | 0.875 | 10000 | 59.0 | 1.0148 | 0.947 | 0.961 |
| | | 3000 | 31.0 | 1.0282 | 0.930 | 0.956 |
| | | 2000 | 25.0 | 1.0350 | 0.923 | 0.955 |
| | | 1000 | 17.7 | 1.0494 | 0.913 | 0.958 |
| | | 500 | 12.5 | 1.0700 | 0.905 | 0.968 |
| | | 400 | 11.0 | 1.0795 | 0.902 | 0.974 |
| | | 300 | 9.5 | 1.0921 | 0.898 | 0.981 |
| | | 200 | 7.7 | 1.1136 | 0.892 | 0.993 |
| | | 100 | 5.4 | 1.1620 | 0.884 | 1.027 |
| | | 20 | 2.3 | 1.3804 | 0.852 | 1.176 |
| 70 | 0.925 | 10000 | 59.0 | 1.0157 | 0.947 | 0.962 |
| | | 3000 | 31.0 | 1.0298 | 0.930 | 0.958 |
| | | 2000 | 25.0 | 1.0370 | 0.923 | 0.957 |
| | | 1000 | 17.7 | 1.0523 | 0.913 | 0.961 |
| | | 500 | 12.5 | 1.0740 | 0.905 | 0.972 |
| | | 400 | 11.0 | 1.0841 | 0.902 | 0.978 |
| | | 300 | 9.5 | 1.0974 | 0.898 | 0.985 |
| | | 200 | 7.7 | 1.1201 | 0.892 | 0.999 |
| | | 100 | 5.4 | 1.1713 | 0.884 | 1.035 |
| | | 20 | 2.3 | 1.4022 | 0.852 | 1.195 |
| 80 | 0.980 | 10000 | 59.0 | 1.0166 | 0.947 | 0.963 |
| | | 3000 | 31.0 | 1.0316 | 0.930 | 0.959 |
| | | 2000 | 25.0 | 1.0392 | 0.923 | 0.959 |
| | | 1000 | 17.7 | 1.0554 | 0.913 | 0.964 |
| | | 500 | 12.5 | 1.0784 | 0.905 | 0.976 |
| | | 400 | 11.0 | 1.0891 | 0.902 | 0.982 |
| | | 300 | 9.5 | 1.1032 | 0.898 | 0.991 |
| | | 200 | 7.7 | 1.1273 | 0.892 | 1.006 |
| | | 100 | 5.4 | 1.1815 | 0.884 | 1.044 |
| | | 20 | 2.3 | 1.4261 | 0.852 | 1.215 |

The complete results of the application of the Arnold theory are given in Table A4 for five different wet-bulb temperatures and for 10 air velocities at each wet-bulb temperature. In Table A5

TABLE A5

| Wet-bulb temperature, $t'$, F | Velocity of air when $t' = t_2$, $V$, fpm |
|---|---|
| 40 | 125 |
| 50 | 145 |
| 60 | 167 |
| 70 | 195 |
| 80 | 235 |

are given the velocities at five different wet-bulb temperatures, where application of the Arnold theory predicts an equality of wet-bulb and adiabatic-saturation temperatures.

# Appendix 2

## Correlation of Experimental Results

*Carrier Experiments of 1911.* Experiments on the effect of radiation upon the reading of a wet-bulb thermometer were performed by Carrier and reported in Appendix 1 of the 1911 paper (1). In these experiments, one wet-bulb thermometer was placed in a tube with an evacuated annular space; all surfaces of this tube were kept wet. The same air was circulated over this wet-bulb thermometer, which was shielded from radiation effects, and over an unshielded wet bulb. The differences between the readings of these two wet-bulb thermometers were reported as a percentage of the wet-bulb depression for six different air velocities.

*Carrier-Lindsay Experiments of 1924.* The results secured by Carrier and Lindsay (8) with the adiabatic saturator described in Appendix 3 of their 1924 paper (8) are not presented here, since Dropkin's results (3) with the same apparatus are believed to be more accurate. However, the apparatus described in Appendix 4 of the same paper (8) gives a good indication of the effects of velocity upon the reading of the wet-bulb thermometer, since simultaneous observations of wet-bulb depressions at like air conditions and different velocities are obtained.

*Dropkin Experiments of 1936.* Dropkin (3) observed the deviation of the wet-bulb temperature from the temperature of adiabatic saturation at several different velocities with the adiabatic saturator used in the Carrier-Lindsay experiments. Due to improvements in the apparatus, his results are probably more accurate than the original ones.

*Arnold Experiment of 1933.* Arnold secured one experimental point for the water-air system. His experiment was performed with dry air at one velocity only. The psychrometer was a side-arm U-tube; the dried air entered the U-tube through the side arm, passed through the U-tube to the annular space between the wet-bulb thermometer and the tube wall and left through the second side arm.

In the correlation of the experimental results, Dropkin's results were plotted directly on a coordinate system with air velocity as abscissa and wet-bulb temperature at entrance to saturator minus wet-bulb temperature at exit from saturator divided by entering wet-bulb depression as ordinate. Only those results of Dropkin's with wet-bulb depressions less than 22 F were used, and the deviations at one velocity for several wet-bulb depressions were averaged arithmetically. These results are shown in Table A6.

TABLE A6

| Air velocity, fpm | $100\left(\dfrac{\text{wet-bulb temp—ad-sat. temp}}{\text{initial wet-bulb dep.}}\right)$ |
|---|---|
| 2000 | −0.88 |
| 1750 | −0.74 |
| 1500 | −0.69 |
| 1250 | −0.66 |
| 1000 | +0.19 |
| 750 | +1.03 |

The results at the higher velocities are probably more reliable because they are more consistent. At the lower velocities, discrepancies between the deviations at two different wet-bulb depressions and the same velocity are very great. For example, at 750 fpm, the deviation is —0.14 per cent for a wet-bulb depression of 7.16 F, and +1.12 per cent for a wet-bulb depression of 10.72 F and practically the same wet-bulb temperature. Therefore, little faith can be placed in the average result at 750 fpm; there is some reason to believe that the lowest experimental deviation at one velocity may be correct, for any failure to maintain completely wetted surfaces will cause high positive deviations with the adiabatic saturator. Dropkin (3) also found some difficulty in maintaining steady conditions at the low air velocities.

The next step in the correlation was to place the line representing the readings of a wet-bulb thermometer completely shielded from radiation upon the coordinate system. A combination of Equations [8] and [10] in Appendix 1 for the water-air system gives

$$EF = \frac{0.622\, r'\,(e' - e_2)}{C_p\,(t - t')} + \frac{r'}{r_2}\left(\frac{t - t_2}{t - t'}\right) \ldots\ldots [13]$$

For the extremely small differences between the wet-bulb temperature $t'$ and the temperature of adiabatic saturation $t_2$, Equation [13] may also be written in a much simpler form with negligible error. This approximate equation is

$$EF = \frac{t - t_2}{t - t'} = 1 + \frac{t' - t_2}{t - t'} \ldots\ldots\ldots\ldots [14]$$

For the perfectly shielded wet-bulb thermometer, $F = 1.0$, and for this case

$$E = 1 + \frac{t' - t_2}{t - t'},$$

or

$$\frac{t' - t_2}{t - t'} = E - 1 \ldots\ldots\ldots\ldots\ldots\ldots [15]$$

The Arnold theory predicts the values of $E$ for water-air given in Table A3 of Appendix 1. Arnold's one experimental point, however, at an air velocity of 736 fpm, gives $EF = 1.115$. The equivalent value of $E$, if Arnold's radiation factor of 1.174 be used, is 0.95. This point does not fall on the straight line representing a plot of $E$ for water versus $E$ for toluene passing through the points (1, 1) and (2.59, 0.83). Instead, this experimental point agrees very well with the value of $E$ that might be calculated from Dropkin's results for this air velocity. Because of this close agreement, the method used for obtaining $E$ was to draw a straight line through Arnold's experimental point on his plot of $E$ for water versus $E$ for toluene and the point (1, 1). The velocity scale obtained by Arnold for his tests with chlorobenzene and toluene was then used to find $E$ for water at different velocities with the results shown in Table A7. Plotting those results gives the line for the shielded wet-bulb thermometer shown in Fig. 1. Quantitatively, these results indicate that, at a velocity of 1000 fpm, the shielded wet-bulb thermometer reads a temperature 4.9 per cent of the wet-bulb depression lower than the true temperature of adiabatic saturation for the water-air system.

TABLE A7

| Air velocity, fpm | Modified Arnold, $E_{water}$ | Wet bulb shielded $\frac{t'-t_2}{t-t'} = E - 1$ | Radiation correction (70 F) $F$ | Wet bulb unshielded $\frac{t'-t_2}{t-t'} = EF - 1$ |
|---|---|---|---|---|
| 3000 | 0.961 | −0.039 | 1.0298 | −0.010 |
| 2000 | 0.957 | −0.043 | 1.0370 | −0.008 |
| 1000 | 0.951 | −0.049 | 1.0523 | +0.001 |
| 500 | 0.947 | −0.053 | 1.0740 | +0.017 |
| 400 | 0.946 | −0.054 | 1.0841 | +0.026 |
| 300 | 0.944 | −0.056 | 1.0974 | +0.036 |
| 200 | 0.942 | −0.058 | 1.1201 | +0.055 |
| 100 | 0.938 | −0.062 | 1.1713 | +0.099 |
| 20 | 0.923 | −0.077 | 1.4022 | +1.294 |

The line on the graph representing the unshielded wet-bulb temperature, as predicted by this modified Arnold theory, may then be added by calculating the radiation correction $F$, and finding $EF = 1 + \frac{t' - t_2}{t - t'}$. The calculation of $F$ in Table A8 is for a wet-bulb temperature of 70 F. The Arnold theoretical results given in Appendix 1 predict equality of the unshielded wet-bulb temperature and the temperature of adiabatic saturation at an air velocity past the wet bulb of about 200 fpm at a wet bulb of 70 F. The use of Arnold's experimental point and a modified theory predict this equality at an air velocity of about 1000 fpm. Furthermore, the modified Arnold curve very closely fits the Dropkin experimental points as shown in Fig. 1.

With the curve representing readings of the shielded wet-bulb temperature placed on the original coordinate system, Carrier's results of 1911 which compared readings of an unshielded and a shielded wet-bulb thermometer at different air velocities may be added. These six experimental points are also shown in Fig. 1.

It should be noted that the product $EF$, which equals $1 + \frac{t' - t_2}{t - t'}$, may be obtained directly by experiments using an adiabatic saturator, like those of Dropkin's, or by experiments like those of Arnold's, since the psychrometric coefficient $A$ equals $\frac{M_a C}{M_v r'}(EF)$. To predict the reading of a shielded wet-bulb thermometer, from experimental results on an unshielded one, involves the calculation of the radiation effect, as given by $F$. If the calculated radiation effect is greater than the actual effect, the curve representing readings of the shielded wet bulb will be too low, and any points referred to this curve will also be low.

The Carrier-Lindsay experiments with like air at different velocities may also be used to predict the position of the unshielded wet-bulb line shown in Fig. 1. Although the absolute values of the wet-bulb deviation found in the original experiment are probably not correct, the shape of the original curves of wet-bulb deviation versus air velocity should be. An empirical equation derived to fit this curve for a wet-bulb temperature of 70 F is

$$\frac{t' - t_2}{t - t'} = \left[\frac{1}{1 - \dfrac{1.023}{(V + 20)^{0.675}}}\right] - 1 \ldots\ldots [16]$$

In order to fix the absolute position of this curve, Dropkin's result at 2000 fpm was assumed to be correct. At this velocity, the empirical equation gives +0.61 per cent for the wet-bulb deviation while the experimental result is −0.88 per cent; therefore, the empirical equation is corrected to fit this point as follows

$$\frac{t' - t_2}{t - t'} = \left[\frac{1}{1 - \dfrac{1.023}{(V + 20)^{0.675}}}\right] - 1 - 0.0149 \ldots\ldots [17]$$

The deviations of the wet-bulb temperature may be found at any air velocity from this equation and plotted as shown in Fig. 1. The shape of this curve is believed by the authors to be nearly correct. Dropkin's results at the air velocities of 750 and 1000 fpm are believed to be a trifle high, and the air velocity for equality of the wet-bulb temperature and the temperature of adiabatic saturation is believed to be between 500 and 600 fpm, although more experimental results are needed at these velocities to confirm this belief.

The adiabatic saturator is a satisfactory device for future experiments. Some apparatus changes may be necessary to secure more consistent results at low velocities. Also, thermocouples may be placed in the jacket of the saturator to check up on the assumption of adiabatic conditions. By initially saturating the air supplied to the heater at some known dew point, the state of the air entering the saturator will be known, and more complete data may be obtained in future experiments.

# Appendix 3

## Properties of Water, Ice, and Steam

The molecular weight of dry air depends upon the composition chosen for the mixture of gases called dry air. If the composition of dry air be taken as that given in vol. 1, page 393 of the International Critical Tables, the equivalent molecular weight of dry air is 28.966. If water vapor obeyed the gas laws, the ratio of the density of water vapor to that of dry air at any common pressure and temperature would be 18.016 ÷ 28.966 or 0.6220.

In Table A8 this ratio of densities for saturated steam is shown as calculated from the Goodenough, Keenan, and Keenan and Keyes steam tables.

TABLE A8 SPECIFIC WEIGHTS OF SATURATED STEAM

Specific weight = $\dfrac{\text{density of saturated steam}}{\text{density of dry air at same } p \text{ and } t}$

| Temperature, F | Goodenough | Keenan | Keenan and Keyes |
|---|---|---|---|
| 32 | 0.62239 | 0.62145 | 0.62163 |
| 40 | 0.62249 | 0.62147 | 0.62173 |
| 50 | 0.62262 | 0.62160 | 0.62177 |
| 60 | 0.62270 | 0.62167 | 0.62184 |
| 70 | 0.62248 | 0.62176 | 0.62203 |
| 80 | 0.62244 | 0.62192 | 0.62235 |
| 90 | 0.62289 | 0.62209 | 0.62256 |
| 100 | 0.62307 | 0.62238 | 0.62274 |
| 212 | 0.63077 | 0.63069 | 0.63115 |
| 300 | 0.64750 | 0.64910 | 0.64885 |
| 400 | 0.68956 | 0.69153 | 0.69044 |

If the deviations from the gas law, denoted by the difference between the specific weight of saturated steam and 0.6220, be plotted against pressure on logarithmic cross-section paper, a straight line will be found to fit all of the Goodenough data very closely. The same straight line will fit the Keenan and Keenan and Keyes data very closely at the higher temperatures, but at the lower temperatures these deviations become negative. The equation of this line fitting the Goodenough data is

$$\text{specific weight} - 0.6220 = 0.0013\, p^{0.713} \quad \text{[18]}$$

where $p$ = the pressure of the saturated steam, lb per sq in. abs.

In the preparation of the psychrometric tables included in this paper, the authors used the Keenan and Keyes data throughout for consistency, but they believe that these specific weights are low, and therefore the specific humidities given in the tables for saturated air are probably too low. Any error, however, is probably very small.

It is interesting to note that if two steam tables were thermodynamically consistent, and if each gave approximately the same pressure-temperature relations for the saturated vapor, then the sigma function, as calculated from either table would be the same. This is true regardless of the differences in the values of specific volume or latent heat appearing in the two tables. If the tabular values satisfy the familiar Clapeyron equation, the table giving the greater volume of the saturated steam would also give the greater latent heat; specific humidities calculated from this table would be lower, and the product of specific humidity and latent heat would be exactly the same from either table. The contribution from the latent heat of the vapor to the sigma function of the mixture of dry air and water vapor in Btu per pound of dry air at any temperature and at a total pressure of $P$, lb per sq in. abs, is

$$\left[\frac{53.3\, T}{144\,(P - p)v_g}\right]\left[\frac{144T\,(v_g - v_f)\left(\dfrac{dp}{dT}\right)_{\text{sat}}}{778}\right]$$

where $T$ = the absolute temperature, F; $p$ = the saturation pressure, lb per sq in. abs; $v_g$ = the specific volume of the saturated vapor, cu ft per lb; and $v_f$ = the specific volume of the saturated liquid, cu ft per lb.

The specific volume of the liquid is insignificant in comparison with that of the vapor at the temperatures involved. Consequently, the contribution of the latent heat to the sigma function of the mixture at any temperature depends only on the saturation pressure and the instantaneous rate of change of that pressure with temperature.

Although the value of specific humidity will depend upon whose steam tables are used in the calculation, the values of sigma function are substantially the same when calculated from the Keenan and Keyes tables as when found from the Goodenough Steam Tables as shown in Table A9.

TABLE A9 SIGMA FUNCTIONS CALCULATED FROM DIFFERENT STEAM TABLES

| Wet-bulb temperature, F | Total pressure = 29.92 in. Hg: Specific humidity for saturation, grains per lb, Goodenough | Keenan and Keyes | Sigma function, Btu per lb of dry air, Goodenough | Keenan and Keyes |
|---|---|---|---|---|
| 32 | 26.47 | 26.37 | 11.73 | 11.73 |
| 40 | 36.41 | 36.34 | 15.16 | 15.16 |
| 50 | 53.47 | 53.39 | 20.12 | 20.13 |
| 60 | 77.3 | 77.26 | 26.09 | 26.10 |
| 70 | 110.5 | 110.3 | 33.41 | 33.40 |
| 80 | 155.8 | 155.6 | 42.53 | 42.51 |
| 90 | 217.6 | 217.4 | 53.99 | 53.99 |
| 100 | 301.3 | 301.0 | 68.62 | 68.60 |

For the properties of mixtures at temperatures below 32 F, the saturation pressures of ice were taken from the new Keenan and Keyes tables which give such values down to —40 F. The pressure of subcooled water was then calculated from the equation given in the International Critical Tables, which is

$$\log \frac{p_w}{p_i} = \frac{-1.1489\,(t - 32)}{459.6 + t} - 0.4105\,(10)^{-5}\,(t - 32)^2 + 1.558\,(10)^{-8}\,(t - 32)^3 \quad \text{[19]}$$

where $p_w$ = the pressure of the subcooled water; $p_i$ = the pressure of saturated ice, in the same units as $p_w$; $t$ = the temperature of the subcooled water, F.

The latent heats of sublimation were also taken from the Keenan and Keyes tables. The latent heats of sublimation of ice may also be found by the Clapeyron relation, after differentiating the equation for the saturation pressure of ice given in the I.C.T. A comparison of these latent heats is given in Table A10.

TABLE A10 LATENT HEAT OF SUBLIMATION OF ICE, BTU PER LB

| Temperature, F | I.C.T. | Keenan and Keyes |
|---|---|---|
| 32 | 1215.3 | 1219.1 |
| 0 | 1217.3 | 1220.7 |
| −10 | 1218.0 | 1221.0 |
| −20 | 1218.7 | 1221.2 |
| −30 | 1219.5 | 1221.2 |
| −40 | 1220.3 | 1221.2 |

The latent heats of vaporization for the subcooled water were found by subtracting from the enthalpy of the saturated vapor, given in the Keenan and Keyes tables, the enthalpy of the subcooled water, calculated from the empirical equation $(t - 32)$.

In preparing the low temperature tables, the Poynting effect was ignored. It is known that the pressure of a liquid (or solid) in contact with a neutral gas and its own vapor is slightly higher than that of the liquid in contact only with its own vapor. This effect for water and ice below 32 F is given in the International Critical Tables as

$$\frac{\Delta p}{p} = \frac{0.36}{459.6 + t} \quad \text{[20]}$$

where $\Delta p$ = the increase in pressure of water or ice due to the presence of air, same units as $p$; $p$ = the normal vapor pressure; and $t$ = the temperature of the water or ice, F.

The Poynting effect causes an increase in pressure (and, also of specific humidity) of about 0.086 per cent at —40 F,

with smaller percentage effects at higher temperatures, and may, therefore, be safely ignored.

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# The Contact-Mixture Analogy Applied to Heat Transfer With Mixtures of Air and Water Vapor

By W. H. CARRIER,[1] NEWARK, N. J.

**The author derives and discusses the general contact-mixture formula for representing physical processes of heat transmission and fluid friction, and points out that the contact-mixture analogy serves directly and logically to correlate heat transfer with fluid friction. He compares the analogy with the conduction-viscosity theory and relates why the contact-mixture analogy explains all the phenomena connected with gas flow and heat transmission.**

HERETOFORE in literature, it has been customary from the time of Reynolds to analyze heat transfer and resistance to flow of gases by using an analogy to the flow of viscous fluids. While this method can be made to give a fairly good correlation of the phenomena of heating and frictional resistance of gases it is not representative of the actual physical process, and it is not easily applied, for example, to condensing and evaporating of water vapor into air except by the application of another analogy. The author believes it to be quite in line with modern physical thought to state that there can be no such thing as shear in a gas and therefore there can be no true viscosity. Also, probably, there is no true conduction in a gas as in a solid but only diffusion of molecules continuously in motion.

Any gas is made up of molecules having different velocities, i.e., different temperatures. The energy of the molecule (i.e., the absolute temperature) varies directly as the square root of its molecular velocity. Therefore, the average temperature of a gas composed of molecules having different molecular velocities is the average of the square roots of their respective velocities. We do not know actually what occurs when a gas contacts with a surface at a different temperature. We do know, however, that molecules in contact with a hotter surface are heated, i.e., their molecular velocity is increased and these high-velocity molecules are diffused and are mixed with other molecules of the gas which have not been so contacted. Whether they retain their identity as high-velocity molecules (which is to be doubted) or whether they impart a portion of their surplus energy to the adjoining molecules, which do not contact the surface, is immaterial as far as any study on heat transmission is concerned. The average of the square roots of the velocity, i.e., the total energy is the same whether they retain their energy or whether they impart part of their increased energy to other molecules.

In the process of pure heat conduction in gases, there is no mechanical mixture or disturbance due to gravitational effect (convection) but only intermolecular diffusion, which depends upon the various permanent properties of the gas and its transient condition. The rate of heat diffusion, however, is found experimentally, to obey exactly the analogous laws of heat conduction, that is, it is directly proportional both to the distance and to the temperature difference between two boundaries. In a steady state of heat flow, there is a temperature gradient precisely as there is a temperature gradient in a solid, although in the first case, the temperature gradient is due solely to material transportation, while in the second case, it is due to the passage of heat from one molecule to another.

These rather obvious and elementary statements are made in the preface in order that there may be no misunderstanding of the basis on which the problem is approached.

When a gas is forced to pass over a surface at relatively high velocity, as for example, between plates or through a pipe, the main stream of air is turbulent above certain critical velocities. However, at all velocities there are two nonturbulent films. The first, which is probably ultramicroscopic or molecular in thickness, is necessarily a dense film of adsorbed gas having approximately the density of liquid or, as some physicists claim, even greater than that of the liquid. This would appear to be a rather permanent film. The second, is a film or zone in which there is a laminar flow as distinguished from a turbulent flow, i.e., all the particles are moving in parallel lines. There is no mechanical mixture within this film. Particles pass from the surface film through the laminar film only by diffusion and heat is conducted only by the process of diffusion, just as though there were no motion whatever within the film since the actual motion is at right angles to the effective path of molecular diffusion. In this film, warmed (contacted) particles pass outward by diffusion while cold (uncontacted) particles, and also previously contacted particles, diffuse inward from the outer surface of the laminar film to the surface film. There is no sharp demarcation between the laminar nonturbulent film and the outer turbulent body of air, but a gradual increase of turbulence. However, from the standpoint of analysis it is convenient to consider a line of sharp demarcation. The thickness of the adsorbed film does not change with the velocity. The thickness of the laminar film, however, varies directly with the velocity and at a somewhat lower rate, i.e., at a fractional power of the velocity.

There is a definite temperature gradient in this laminar film

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and there is also what amounts to a temperature gradient in the adjacent turbulent film as shown by Arnold[2] and referred to in a paper by Carrier and Mackey.[3]

The author believes this is a correct statement of the actual physical processes which exist in relation to heat transfer and to frictional resistance to gaseous flow.

The expression of relationships through the contact-mixture analogy or theory, seems to the author to represent much more closely the actual physical processes both for heat transmission and for friction than does the quite far-fetched conduction-viscosity analogy. For practical engineering purposes it is also more valuable than the latter. The possible exception to this is where there is a secondary process of heat transfer in series. Then, for a correct solution, it is necessary to employ the resistance concept to combine both processes. This transformation, however, is readily made as indicated in the (resistance) formula. The analysis itself is so simple as to be almost obvious and in fact it is a method used (in effect) many years by those employed in manufacture of heating and air-conditioning equipment.

The contact-mixture analogy involves four approximate assumptions:

1 That in any heat-transfer system, whether adiabatic or nonadiabatic, there is a definite hypothetical surface temperature which is approximately that of the actual surface temperature and which can be evaluated experimentally.

2 That certain particles of the air stream come in actual contact with this surface and are brought to a temperature and a moisture content corresponding to that of the surface film.

3 That for any unit of a heat-transfer system, there is an equivalent of a definite percentage of physical contact of the particles of the air stream with the actual surface. This varies with the conformation of the surface and with the velocity, which variation can be determined experimentally.

4 On the basis of the three previous assumptions, the process of heat and moisture transfer may be considered purely one of mechanical mixture, i.e., a mixture of a percentage of contact air with the remainder of uncontacted air.

This method nevertheless gives remarkably accurate and consistent results when the two hypothetical factors of surface temperature and per cent of contact are once evaluated, by experiment, for a given type of surface. These same experiments have to be made with any type of surface or any system of heat-transfer calculations regardless of what theory is applied, therefore, the same experimental work is involved in any case, but the simplification of the results of this approximate and empirical basis has been found to give not only extremely accurate results for all applications, but greatly simplifies all subsequent engineering calculations through a very wide range of applications.

It has been applied with success in engineering to (*a*) heating and cooling of air with heat-transfer surfaces where no change in moisture content occurs; (*b*) calculation of adiabatic-saturation processes involving humidification and evaporative cooling; (*c*) calculation of the performance of cooling towers; (*d*) calculation of the performance of "evaporative condensers" such as employed in refrigeration; and (*e*) calculation of cooling and dehumidifying of moist air by contact with cold surfaces, as in cooling coils and heat interchangers in air conditioning. It also has other practical applications.

It is further interesting to note that if the constants be determined for one of the processes, they can be employed with sufficient commercial accuracy to predict the result with each of the remaining processes. The proof of the accuracy, and therefore the value of this correlation, can only be demonstrated by actual employment and test of results obtained. This had been done with gratifying success.

In this connection, the common assumption that the evaporation of water into air is basically proportional to vapor-pressure differences, should be contradicted. The process is partly one of mechanical mixture and partly one of molecular diffusion. The rate of diffusion depends on the relative volumetric proportions of air and water vapor present in the mixture; in other words, upon the relative number of molecules of each in the mixture and not upon the vapor pressure. The vapor pressure in itself, does not directly cause diffusion as is commonly assumed. Indirectly, of course, it is mathematically related solely for the reason that the number of molecules present in a mixture is proportional to the hypothetical partial pressures and that the temperature-pressure relationship of water vapor definitely determines the maximum possible pressure of water vapor which can exist in a mixture. The vapor pressure itself has nothing to do directly with the process of diffusion, as engineers often assume, i.e., it is not the direct motivating force, although vapor pressure, partial pressure and molecular diffusion are each the result of molecular activity, defined as temperature. This concept is important in an understanding of the contact-mixture theory in evaporation and condensation.

## Derivation of the General Contact-Mixture Formula

Assume a fluid passing over a surface $S$ at a constant rate of flow $\frac{dQ}{dt}$ through a corresponding area $A$ as shown in Fig. 1. If the flow is turbulent, we may assume that an equal number of fluid

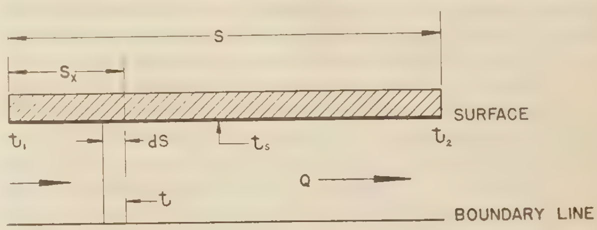


Fig. 1

particles $\Delta Q_\Sigma$ contact each unit of surface $\Delta S$ in a unit of time and are mixed uniformly with the remaining fluid $Q - \int \Delta Q$ (i.e., $Q - Q_{cx}$). We may further assume that each contacting particle acquires a state corresponding to the surface and retains this state throughout the entire process, also that the uncontacted particles retain their original state throughout the process; i.e., $Q$ at any point $x$ is a homogeneous mixture of contacted particles ($Q_{cx}$) and uncontacted particles ($Q - Q_{cx}$). On this empirical hypothesis the number of new contacts with a surface element is in the ratio

$$\frac{\Delta Q_{cx}}{\Delta Q_\Sigma} = \frac{Q - Q_{cx}}{Q} \qquad [1]$$

Since the number of total contacts $\Delta Q_\Sigma$ per element of surface $\Delta S$ is assumed constant for any fixed condition of flow and character of surface, we may define this constant $m$ as

$$m = \frac{\Delta Q}{\Delta S} = \frac{dQ_\Sigma}{dS} \qquad [2]$$

but combining Equations [1] and [2]

$$\frac{dQ_{cx}}{Q - Q_{cx}} = \left(\frac{m}{Q}\right) dS \qquad [3]$$

[2] "The Theory of the Psychrometer," by J. H. Arnold, *Physics*, vol. 4, July and September, 1933, p. 255 and p. 334.

[3] "A Review of the Existing Psychrometric Data in Relation to Practical Engineering Problems," by W. H. Carrier and C. O. Mackey, Trans. A.S.M.E., vol. 59, 1937, paper PRO-59-1, p. 33.

Integrating between the limits

$$Q_{cx}\begin{cases} = 0 \\ = Q_c \end{cases} \text{and } S_x \begin{cases} = 0 \\ = S \end{cases} \text{we have}$$

$$\log_e \frac{Q}{Q - Q_c} = \left(\frac{m}{Q}\right) S \ldots\ldots\ldots\ldots [4]$$

$$\log_e \left[\frac{1}{\left(1 - \frac{Q_c}{Q}\right)}\right] = \left(\frac{m}{Q}\right) S \ldots\ldots\ldots\ldots [5]$$

$$1 - \frac{Q_c}{Q} = e^{-mS/Q} \ldots\ldots\ldots\ldots\ldots\ldots [6]$$

The ratio $\left(1 - \frac{Q_c}{Q}\right)$ is aptly termed by those employing this method as the "by-pass factor" since it represents the ratio of uncontacted fluid to the total quantity or the equivalent of the proportion by-passed if the remainder be considered subjected to a theoretically perfect contact. Usually the contacting surface consists of a series of repetitive units of like geometrical proportions through which the gas (air) passes in succession as for example, rows of staggered pipe or finned tubing or checker work (in a forced-draft cooling tower, chemical absorber or gas cleaner), or even drops of water. If the surface in the unit is $S_u$, and the number of units in a series in the path of the fluid flow is $n$, then Equation [6] becomes

$$\left(1 - \frac{Q_c}{Q}\right) = \left(e^{-mS/Q}\right)^n = \left[\left(1 - \frac{Q_c}{Q}\right)_u\right]^n \ldots\ldots\ldots [7]$$

That is, the by-pass factor for $n$ units in series is the $n$th power of the by-pass factor for one unit.

In the case of heat interchange or other process in which there is a surface equilibrium established, the possible change in surface condition must be taken into account as well as the element of contact.

Equation [6] may be considered the fundamental equation of the contact-mixture analogy. In form and method of analysis it is the same as that employed by the author in his 1911 paper[4] on the theory of heat transfer from surface to air in heating, and in cooling and condensing. However, he transformed this into an artificial resistance and conductance concept. The foregoing relation, although in part artificial as are all other methods of approach nevertheless is often a convenient general and fundamental solution to commercial problems of heating, cooling, evaporation and condensation of vapors in gaseous mixtures. For combinations other than water and air, however, Arnold's correction ratio for differences in diffusion rates, which is discussed in a paper[2] by Carrier and Mackey, will have to be employed. For this purpose Arnold's study is a most valuable contribution. In general, his theory is obviously correct but we believe his correlation could be simplified by the use in part of the contact-mixture concept instead of the Prandtl analogy. The contact-mixture analogy also applies more logically to theory of gaseous fluid friction than does the Reynolds' theory, since there is actually no shear in gases and therefore no true viscosity, but only loss and gain of momentum of molecules in the processes of contact, molecular diffusion, and mechanical mixture. Here mechanical mixture and consequent change of momentum are the preponderant factors at higher velocities in determining energy losses. By analogy, mechanical mixture would seem to be the most important factor in processes of heat transfer at these velocities.

The contact-mixture analogy may also serve to correlate directly and logically, heat transfer with fluid friction. It offers perhaps a more basic approach than previous methods.

The following examples will illustrate the use of the contact-mixture analogy:

*Example No. 1.* Assume air to be heated with a series of heating elements in which $m$ has been determined by experiment for a given air velocity for a given type of surface in accordance with Equation [7] where $n = 1$. Assume the surface of these elements have the constant average temperature $t_s$ and that the initial temperature of the air is $t_1$. What is the final temperature $t_2$? The percentage of noncontacting air, i.e., by-passed air is

$$\left[\left(1 - \frac{Q_c}{Q}\right)_u\right]^n = \left(e^{-m/S_u}\right)^n$$

This will have its original temperature $t_1$.

The percentage of contacting air is obviously

$$1 - \left[\left(1 - \frac{Q_c}{Q}\right)_u\right]^n = \left(e^{-mS_u}\right)^n = 1 - \left(e^{-mS_u}\right)^n$$

This portion is at the temperature $t_s$.

The temperature of the mixture is obviously

$$t_2 = \left[\left(1 - \frac{Q_c}{Q}\right)_u\right]^n t_1 + \left[1 - \left\{\left(1 - \frac{Q_c}{Q}\right)_u\right\}^n\right] t_s$$

$$t_2 = \left(e^{-mS_u}\right)^n t_1 + \left[1 - \left(e^{-mS_u}\right)^n\right] t_s$$

For example, if $t_1 = 20$ F, $t_s = 227$ F, $e^{-mS_u} = 0.70$, and $n - 4$, then we will have

$$t_2 = [(0.70)^4 \times 20] + [1 - (0.70)^4] \times 227]$$

$$= (0.24 \times 20) + (0.76 \times 227) = 177.3 \text{ F}$$

*Example No. 2.* Assume that the same surface is used in cooling and dehumidifying air and that the refrigerant temperature in cooling coils is 40 F, and that the mean surface temperature is 46 F (this varies with the load); that $t_1 = 80$ F, and that the corresponding relative humidity is approximately 50 per cent, i.e., that there are 78 grains of moisture per pound of dry air ($W_1 = 78$).

Taking $t_s$ as 46 F, $W_s = 46$ grains, the by-pass factor is approximately the same as before, i.e., 0.24. The final temperature $t_2$ is

$$t_2 = (0.24 \times 80) + (0.76 \times 46) = 54.2 \text{ F}$$

The moisture content $W_2$ is

$$W_2 = (0.24 \times 78) + (0.76 \times 46) = 53.7 \text{ grains}$$

Therefore, the final condition of the air is 54.2 F and contains 53.7 grains of moisture, approximately. Referring to the psychrometric chart, the dew point is approximately 50 F, the wet-bulb temperature slightly under 52 F, and the relative humidity approximately 87 per cent for the leaving air. Moisture calculations are on the assumption of standard barometric conditions.

It should, however, be noted that it is possible to obtain the same result directly from the chart and the by-pass factor, by drawing a line connecting the point corresponding to 80 F and 78 grains of moisture to the point corresponding to 46 F saturated. Then taking a point on the line located at 24 per cent of the length of the line from the saturation point, i.e., taking the intersection of this line with the abscissa 8.2 = [0.24 (80 F — 46 F)] from the 46 F abscissa as shown in Fig. 2, we get

[4] "Air-Conditioning Apparatus," by W. H. Carrier and F. L. Busey, Trans. A.S.M.E., vol. 33, 1911, p. 1055.

graphically the same results of temperature and moisture content as before. This necessarily follows from the fact that on the chart the grains of moisture are represented as linear functions of temperature. This line drawn on the chart represents the cooling path of the air with respect to both temperature and moisture as it passes over the surface. The value of employing the chart is to show graphically this linear relationship. This is a common practice. But it depends entirely upon the approximate validity of the contact-mixture analogy. Actually, the ratio in change of moisture content should be slightly greater

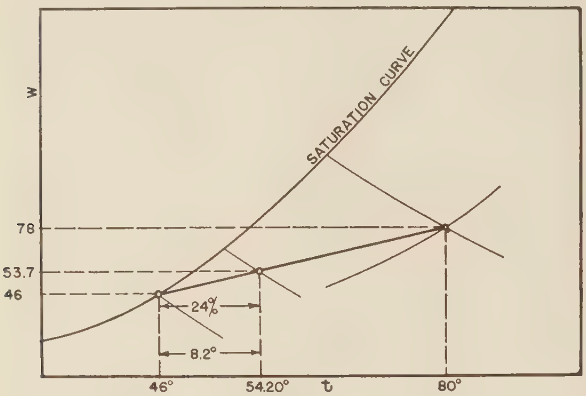


FIG. 2

than the corresponding ratio of temperature change owing to the fact that the rate of moisture diffusion in the laminar film is greater than that of the accompanying air. The error will be about in the order of the difference between the temperature of adiabatic saturation and the radiation-shielded wet bulb. This is, as shown in another paper[3] on psychrometry, somewhere in the order of 3 per cent for water vapor and air at high velocities. This error would hardly be observable commercially.

A corresponding process of calculation can be employed in determining the results to be obtained from evaporative condensers. In both condensing and evaporating surface, it is usual to employ nonferrous extended surface coils and the difference between the refrigerant temperature and surface temperature is a linear function of the total heat load. This factor has to be determined experimentally for each general type of heater. Experimental determinations check out within the accuracy of observation, the dependability of the foregoing method of calculation.

### Comparison of Contact-Mixture Analogy With Conductance Formula

It is further interesting to compare the general equation of the contact-mixture analogy with that obtained by the use of the thermal-resistance or thermal-conductance concept. From the former analogy, we have the following equation wherein $m$ contact factor, $S$ = the total surface, $t_s$ = the surface temperature, $t_1$ = the initial temperature of air, and $t_2$ = the final temperature

$$t_2 = (e^{-mS/Q})\ t_1 + (1 - e^{-mS/Q})\ t_s \qquad [8]$$

By rearrangement

$$\frac{t_s - t_1}{t_s - t_2} = e^{-mS/Q} \qquad [9]$$

$$\log_e \left(\frac{t_s - t_1}{t_s - t_2}\right) = -\left(\frac{m}{Q}\right) S \qquad [10]$$

From the conductance concept, we have the general formula where $K$ is the coefficient of conductance in Btu per square foot per hour of surface per degree difference of temperature; $dH$ is the differential rate of heat transfer per unit of time and $dS$ is the differential of surface. From the thermal properties of the air, where $C_p$ is the specific heat of the air; $G$ is the total weight in pounds of air per hour flowing over the element of surface $dS$; $dt$ is a change of temperature which takes place in the air when flowing over the element of surface $dS$ at the rate of $G$; and $K$ is the coefficient of heat transfer between the surface and the air for the particular condition, we have the following two equations for $dH$

$$dH = K\,(t_s - t)\,dS$$

also

$$dH = C_pG\,dt$$

Therefore, we may equate the two equivalent values of $dH$ as follows

$$C_pG\,dt = dH = K\,(t_s - t)\,dS$$

$$\frac{dt}{t_s - t} = \frac{K}{C_pG}\,dS$$

Integrating this equation between the limits

$$t \begin{cases} = t_1 \\ = t_2 \end{cases} \text{and } S \begin{cases} = 0 \\ = S \end{cases}$$

we have

$$\log_e \left(\frac{t_s - t_1}{t_s - t_2}\right) = \left(\frac{K}{C_pG}\right) S \qquad [11]$$

It will be noted that Equation [11] is identical in form with Equation [10] and

$$\frac{m}{Q} = \frac{K}{C_pG} = \frac{K}{C_p\rho Q}$$

hence

$$m = \frac{K}{C_p\rho}$$

It is to be noted that $K$ is an experimental value dependent on the velocity, and is also a direct function of the specific heat $C_p$ and the density $\rho$; $G$ is the weight of the air in pounds per hour and is a direct function of the velocity, the free area through the heater and the density of the air. For fuller discussion of these relationships refer to the author's 1911 paper[4] and a paper[5] by Lewis, McAdams, and Frost.

### Frictional Resistance to Flow

It is almost obvious that the same reasoning will apply with equal force to determination of frictional resistance to flow of gases and to the correlation of the frictional resistance with heat transfer. Consider as before, a body of air $Q$ passing over a surface $S$. In an element of surface $dS$ there will be a definite contact as before of $\Delta Q_\Sigma$ particles with the element of surface. Let $m$ be this ratio. Then as before, we have $\Delta Q_\Sigma = m\Delta S$ also

$$Q_\Sigma = mS \qquad [12]$$

but combining with Equation [4], we have

$$\frac{Q_\Sigma}{Q} = \log_e \left[\frac{1}{1 - \left(\frac{Q_c}{Q}\right)}\right] \qquad [13]$$

[5] "Heat Transfer by Conduction and Convection," by W. K. Lewis, W. H. McAdams, and T. H. Frost, Trans. A.S.H.&V.E., vol. 28, 1922, p. 55.

which means that the ratio of the total number of particles coming in contact with a surface, to the total volume handled is in the ratio of the logarithm of the reciprocal of the by-pass factor. It is assumed that each particle that actually contacts with the surface film loses its momentum completely which has to be restored upon its diffusion through the film and mechanical mixture into the body of the gas stream.

Let $E_f$ be the energy absorbed by gaseous friction (i.e., by loss of momentum of the particles contacted with the surface), $u$ the velocity of the gas stream, and $\rho$ the density of the gas, then

$$E_f = \frac{Q\Sigma\rho u^2}{2g} = \frac{Q\rho u^2}{2g}\log_e\left[1 - \frac{1}{\left(\frac{Q_c}{Q}\right)}\right] \quad \text{.........[14]}$$

The term, "velocity head" is quite generally employed to mean the head in terms of a gas (or equivalent liquid) column corresponding to the velocity. Let this be designated by $h_u$ which is defined as

$$h_u = \frac{u^2}{2g}$$

but from the general energy equation $E_f = Q\rho h_f$ therefore

$$\frac{h_f}{h_u} = \log_e\left[1 - \frac{1}{\left(\frac{Q_c}{Q}\right)}\right] = \frac{mS}{Q}$$

which means that the number of velocity heads lost is equal to the logarithm of the reciprocal of the by-pass factor,

$$\frac{h_f}{h_u} = \log_e\left(\frac{1}{1-\frac{Q_c}{Q}}\right) = \frac{mS}{Q} = \frac{KS}{C_pG} = \log_e\left(\frac{t_s - t_1}{t_s - t_2}\right)$$

$$= \log_e\left(\frac{W_s - W_1}{W_s - W_2}\right)$$

These equations it will be seen tie together all relationships basic to the contact-mixture theory. As previously explained, however, the formula for change in moisture content is only approximately correct for the reason that the value $m$ for water vapor is slightly greater than that for the accompanying air owing to its greater rate of diffusion in the laminar film. At high velocities this difference is of little significance while at very low velocities it may become very appreciable. In most commercial installations, however, the velocities are high rather than low.

### Application of Contact-Mixture Theory to Nonturbulent Flow

From the discussion on turbulent flow it is evident that the friction varies as a power of velocity slightly less than 2 because of the fact that $m$ does not increase quite as fast as the velocity. With the nonturbulent flow, however, there is still contact with the surface film but no mechanical mixture. Here it can be shown, by the same reasoning, that the friction is approximately directly proportional to the velocity instead of nearly proportional to the square of the velocity.

In nonturbulent or laminar flow, which occurs at velocities below the critical, the heat conduction is not increased by increase of velocity, i.e., the number of contacts is not increased by any change of velocity since the particles are mixed entirely by molecular diffusion and not by mechanical mixture. In the case of turbulent flow $m$ increases with the velocity at a power slightly less than unity. If we represent $m$ for turbulent flow as $m_n$, we will have the approximate ratio of

$$\frac{m_n}{m_T} = \frac{\gamma}{u}$$

or

$$m_n = \frac{m_T\gamma}{u}$$

From this it will be seen that if the friction with turbulent flow varies approximately as the square of the velocity, then the friction of nonturbulent flow varies directly as the velocity.

It will be seen from all the foregoing relationships that the contact-mixture theory adequately explains, at least by analogy, all the phenomena connected with gas flow and heat transmission. Also, that it conforms with the accepted dynamical theory of gases.

# The Contributions of the U. S. Bureau of Mines to Helium Production[1]

By C. W. SEIBEL,[2] AMARILLO, TEXAS

**The production of helium for commercial use is of particular interest at this time since it had its inception in the State of Texas and has been carried on chiefly in that state ever since. The development of helium production is an excellent example of how American engineers attempt the seemingly impossible—and win. It also demonstrates the rapidity with which an almost hopeless undertaking can be completed when a determination really has been made to do so and the necessary funds are available. In this paper are outlined the early history of helium production, the steps in the necessary preliminary research, and how the data were applied. Early repurification plants are described, and facts given concerning the present Amarillo plant.**

IN APRIL, 1917, when America entered the World War, helium was a chemical curiosity. Radioactive minerals were the chief source of supply; not more than a hundred cubic feet of helium had been isolated in the world and probably not more than two or three cubic feet in the United States.[3] It sold in small quantities at the rate of $2500 per cu ft. At that rate it would have cost approximately $450,000,000 to produce enough helium to fill a small blimp.

The British Government was anxious to replace hydrogen in lighter-than-air craft with an inert medium and helium seemed to be the only possible substitute. Disregarding the apparent hopelessness of the undertaking, it was suggested by English scientists, headed by Sir Richard Threlfall, Sir William Ramsey, and others, that the production of helium for aeronautical purposes be investigated. The matter came to the attention of Dr. R. B. Moore and G. A. Burrell, both of the U. S. Bureau of Mines; they called upon an official of the War Department on June 1, 1917, and explained the possibilities to him. On that date our present story really began.

Matters moved rapidly in those days, and by July 31, 1917, $100,000 had been allotted in equal shares by the Army and the Navy to investigate the possibility of producing helium as a substitute for hydrogen for balloons and dirigibles.

The undertaking naturally fell into two major divisions: What to use as the source of helium, and the method to be employed to extract it from the raw material.

Through discoveries by Dr. H. P. Cady and David McFarland of the University of Kansas, in 1905, it was known that certain natural gases contained appreciable amounts of helium. A search was started at once for a suitable gas field.

Dr. Cady was appointed consulting chemist on the staff of the Bureau of Mines and all the early analytical work on field samples was performed by Cady and his coworkers. He also engaged in research work, the results of which influenced the design of various helium plants that were constructed later. During the early days Cady contributed materially to knowledge of the limits of inflammability of mixtures of helium and hydrogen, the solubility of helium in liquid natural gas and nitrogen, and the diffusion of helium through balloon fabric.

As a result of the field investigations it was decided to use as the source of helium natural gas from the Petrolia Field of Texas, which, by reason of the Lone Star Gas Company's pipe-line system, was available to the city of Fort Worth. That gas contained approximately one per cent of helium.

It was conceded generally that if helium were to be produced from natural gas the system to be used probably would involve the liquefaction by means of pressure and low temperature of all of the constituents of the natural gas except the helium. An investigation, therefore, was made of the processes employing low-temperature liquefaction in the production of oxygen from liquid air, particularly those of the Linde Air Products Company, the Air Reduction Company, and the Jefferies-Norton Corporation.

Perhaps speed was lent the project by a letter from the English Admiralty received the latter part of July, 1917, in which it requested that America do what it could to produce helium and that it would like to obtain 100,000,000 cu ft at once and would be in a position to take 1,000,000 cu ft per week thereafter. Additional sums were allotted by the Army and Navy in October, 1917, and by November the matter had progressed so far that contracts were awarded the Linde Air Products Company and the Air Reduction Company covering the building of modified units of their liquid-air plants for the production of helium. Later a similar contract was made with the Jefferies-Norton Corporation.

Construction of the two experimental plants on the outskirts of Fort Worth, Texas, was begun almost at once, the work being supervised by the Quartermaster Corps of the U. S. Army. The Linde plant was completed in March, 1918, and the Air Reduction Company's plant in May. Helium of 28 per cent purity was produced by the Linde plant by March 22, 1918. The quantity and quality of the output of this plant improved gradually until by September the production of about 5000 cu ft per day of approximately 70 per cent helium was being maintained. This crude helium was then repurified and a 92 or 93 per cent product was produced. Varying amounts of helium also were produced by the Air Reduction and Jefferies-Norton plants. By November, 1918, only one year after the contract to build the plants had been let more than 200,000 cu ft of helium of 93 per cent purity had

[1] Published by permission of the Director, U. S. Bureau of Mines.

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[3] "Commercial Production of Helium," by F. G. Cottrell, *Chemical and Metallurgical Engineering*, vol. 20, no. 3, Feb. 1, 1919, pp. 104–114.

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been produced and 147,000 cu ft of it was on the dock at New Orleans, La., awaiting shipment to Europe.

American industry had stepped into the breach again. It had accepted the challenge to produce a chemical element that was contained in a natural gas to the extent of only one per cent; a substance that had been a chemical curiosity up to that time; not more than 100 cu ft of which had been isolated in all the world; which had sold for $2500 a cu ft; and for the production of which no commercial-scale plant had ever been contemplated. Less than one year after accepting the challenge, two plants had been constructed and the third one was under way. Helium was being produced in quantities at a purity of 93 per cent and at a cost of approximately 44 cents per cu ft.

During the war the Bureau of Mines had jurisdiction over the experimental plants. Shortly after the armistice these units were closed, but the production of helium was a reality and the Army and the Navy had already decided to build and had plans well along for a full-scale unit with a capacity of about 50,000 cu ft of helium per day. The new unit was to use the Linde system, which had been most successful in the experimental stage. The construction and operation of the new plant was under the jurisdiction of the Navy Department, and the Navy maintained supervision of it until July 1, 1925, when all helium production was placed under the Bureau of Mines. All told, the Fort Worth helium plant produced approximately 48,000,000 cu ft of helium during its existence.

Even during the life of the experimental plants it was recognized that the Petrolia Field could not be depended on as a source of helium for many years; in fact, it was estimated that it would be depleted, in so far as large-scale helium production was concerned, in approximately ten years, or by 1929. However, in a war project other factors more than offset its limited capacity. The prediction was proved to be accurate, and it was seen as early as 1927 that the Petrolia Field could not support the helium project on an economical basis much longer. The plant was shut down in January, 1929, and helium activities were moved to Amarillo, Tex.

## Research

It is not surprising that the experimental helium plant developed during the press of war days was not all that could be desired from the standpoint of efficiency and economy. The engineers of the Bureau of Mines felt that if further thought and experimental study could be given to the production of helium, the cost could be reduced greatly, even below the best anticipated performance of the large Fort Worth plant. Therefore, with the financial assistance of the Army and Navy the Bureau established its Cryogenic Laboratory at Washington, D. C., early in 1921. It obtained a staff of research workers who set out to learn something of the fundamentals required in the efficient production of helium.

The ramifications of the research were wide and varied. There were the questions of specific heats, composition of coexisting phases, solubility of helium in the liquid components of natural gas, behavior of metals at low temperatures, calculation of heat-exchanger surfaces, design of the interchangers, prior removal of certain objectionable constituents from natural gas, such as carbon dioxide, the method of refrigeration to be employed, development of valves for special purposes, and selection of heat insulators. Through it all the Bureau carried on field investigations, hoping for the discovery of other sources of helium supplies. Some of these phases will be considered briefly here.

With the cooperation of the U. S. Navy Yard at Washington, D. C., a rather comprehensive study was made of the behavior of various metals at liquid-air temperature. It was found that metals increase their ultimate tensile strength from a few per cent to nearly 100 per cent at the temperature of liquid air, approximately 312 deg below zero F. However, most of the steels tested lost practically all of their ductility and showed very little if any elongation. Some of the nonferrous metals, such as copper, brass, monel metal, and nickel, retained an appreciable percentage of their normal ductility. It was thought that solder, because of its tin content, might suffer through a change of the tin to the gray modification. However, possibly because low temperature retards the rate of reaction, there has been no evidence, either during experimental work or in actual use, to indicate that solder suffers materially from "tin disease" at liquid-air temperature. As a result of this experimental work it was decided in so far as possible to eliminate the use of steel wherever very low temperatures were involved. As a result virtually all of the low-temperature equipment of the helium-production plant is fabricated either from hard-drawn copper or rolled or worked naval bronze.

At the time the experimental helium plant at Fort Worth was designed, and even when the large Fort Worth plant was designed, little information was available concerning the composition of coexisting phases of the various constituents of our natural gas. Furthermore, there was not time to develop that information. The fact that the experimental plant worked at all speaks well for some of the intelligent guesses that had to be made as to numerous features of the design at that time.

As a result of the work in the Bureau's cryogenic laboratory at Washington, D. C., many missing pieces of information were supplied. Cylinders of Petrolia gas under high pressure were sent to Washington and data were obtained which enabled curves to be drawn showing the composition of the liquid and gas phases under different conditions of temperature and pressure. These data were used in designing the equipment and in selecting the type of cycle to be used.

It was realized early that the solubility of helium in liquefied natural gases would determine almost entirely whether fractionation would be required to obtain satisfactory recovery. The experimental work developed the rather curious fact that above a temperature of about minus 180 F little helium would be lost by solution in the liquid phase under a certain pressure. At a given pressure, as the temperature was lowered, the amount of helium lost through solubility effect, gradually increased until a temperature of about minus 265 F was reached. At still lower temperatures the solubility fell off rapidly until, at the temperature of liquid air, the solubility effect was little more than that at minus 180 F. It will be realized, of course, that these figures are only approximate and vary with the composition of the natural gas. These relations, determined experimentally, indicate that for maximum recovery of helium the liquefied constituents should be separated from the gas phase either at a relatively warm temperature (that is, above minus 180 F), or at a relatively cold one (approximately minus 292 F to minus 310 F).

Little could be learned from the literature relative to the method of calculating heat transfer at these very low temperatures. Experimental work therefore was undertaken on different types of heat exchangers. As a result of these experiments the Bureau has preferred to use, where possible, interchangers of a multiple-tube type. The construction is somewhat like that of a fire-tube boiler, being primarily a series of relatively small tubes soldered into a tube sheet and surrounded by a wall of a thickness necessary to withstand the pressure required. It might be mentioned that most heat-exchanger calculations are based on the use of clean tubes. For the calculation of heat exchangers at very low temperatures one must not fail to consider that the material handled may contain appreciable quantities of water vapor or other condensable substances that may condense to solids on the tubes of the exchanger. These solids, if on the inside of the tube, diminish the heat-exchange surface and reduce the ability

of the tubes to transmit heat. As a result, in designing interchangers for this type of work care must be taken not to let data obtained for clean tubes influence one unduly.

The matter of applying heat-insulating material received considerable thought by the engineers of the Bureau of Mines. Substances that may be adequate for moderately low temperatures may not be entirely satisfactory when the temperature of the equipment to be insulated approaches that of liquid air. Effectiveness of the insulating material from a standpoint of overall economy and efficiency was investigated and decided upon finally to the satisfaction of the Bureau. Two kinds of insulation are used, felted hair and a so-called "mineral" wool. Both have advantages and disadvantages when used in the type of plant under discussion. Insulation varies in thickness, depending upon the type and size of the equipment to be insulated, from about 3 to 10 inches. Because the insulation in the Amarillo plant is adequate, there is little external evidence of the intense cold that is maintained in parts of the equipment.

The Bureau investigated the relative worth of the two systems of low-temperature refrigeration, that obtained by means of an expansion engine and that due to the Joule-Thomson effect. Here again each method has its adherents. While theoretically the use of expansion engines is considerably more efficient than the use of the Joule-Thomson effect, practically, the advantage is perhaps more apparent than real. In the helium plant under discussion refrigeration is obtained by means of both systems, although an expansion engine is used to obtain the major portion.

The helium molecule is very small, therefore pipe work, valves, and other fittings that might be entirely tight against a pressure of several thousand pounds of air might leak appreciably under perhaps one half to one third the same pressure of helium. Because of this and as the Bureau wished to obtain long-stem valves in order that their bodies might be insulated and the heat leak thus reduced as much as possible, several types of valves were designed. A special type of packing for the valve stems also was developed, since packing containing an oily lubricant could not be used at low temperature. Leather seems to be the most satisfactory material to use as valve-stem packing.

Many special instruments and pieces of equipment have been designed and constructed by Bureau research workers in connection with the production of helium. Continuous recorders have been developed to record the percentage of carbon dioxide in the incoming and outgoing gases; also, a somewhat complicated but nevertheless satisfactory piece of equipment has been developed for recording the amount of helium in the tail gas. When it is considered that the tailings contain only about one eighth of one per cent of helium in an otherwise normal natural gas, one can realize the sensitivity of the apparatus.

### Early Repurification Plants

It is evident that helium leaks slowly from the envelope of the airship. It is not so apparent, but nevertheless is true, that air leaks into the envelope and eventually dilutes the helium to such a degree that it is no longer economical to use it as a lifting medium. Even before a helium-filled ship had actually flown, it was appreciated that a repurification plant would be desirable for the purpose of removing the contaminating air and bringing the purity of the helium to 98 per cent or more. The increased purity of approximately 5 per cent over that of experimental days is quite an item in lighter-than-air craft operation. Depending upon the type and size of the ship, this 5 per cent greater purity may amount to a much greater percentage of increase in pay load.

In 1921, when the cost of producing helium was very high, Bureau of Mines engineers designed and constructed a mobile repurification plant using refrigerated charcoal for the removal of air. Helium having a purity of better than 99 per cent was obtained. The plant was abandoned later, however, because of operating complications and the fact that the cost of producing helium from natural gas had been reduced below the cost of repurification by refrigerated charcoal.

At about the same time the Bureau also sponsored a repurification plant based on the liquefaction of the contaminating air. Through an accident this plant was made inoperative before it had had an opportunity to prove itself.

The two plants mentioned above were constructed before adequate experimental data were available and should therefore be classed with the experimental production plants built during the war days.

### Research Data Applied

As a result of much experimental work, such as has been outlined briefly, Bureau of Mines engineers were able to design for the U. S. Navy at Lakehurst, N. J., a helium-repurification plant with a capacity of 20,000 cu ft of helium per hr. Its performance equaled the prediction made for it at the outset and as a result the Bureau was requested to build a similar small mobile plant for the Army.

This mobile unit was self-contained, was mounted on a railroad boxcar, and had a capacity of 5000 cu ft per hr. It has since done good work both at Scott Field and at Langley Field, stations of the Army Air Corps. Later, a stationary plant was designed for the Army and was erected by the personnel of Scott Field, Belleville, Ill. The present capacity of this plant is 10,000 cu ft of helium per hr, although the equipment has been designed so that, with the addition of the necessary helium compressor, the total capacity can be doubled.

While the repurification plants were being designed and constructed, a group of Bureau of Mines engineers was working on the production of helium from natural gas. Their work went through several stages, including laboratory experimental work, a laboratory-size model, a semicommercial-scale model, and eventually a full-sized plant.

### Amarillo Plant

When, in 1927, it was definitely evident that the Petrolia Field could no longer meet the helium requirements of the Army and Navy, the experimental work had progressed to a point where the Bureau felt that it could design a helium plant that would reduce the cost of production materially.

The field investigators of the Bureau had discovered a helium-bearing gas field at Amarillo, Tex. This field was owned in four large blocks, was practically virgin, and the natural gas in it contained approximately $1^3/_4$ per cent of helium. The natural gas in this field is under a rock pressure of more than 700 lb per sq in. The Government now owns the gas rights in fee in 50,000 acres covering the field.

A plant site $7^1/_2$ miles west of the city of Amarillo and connected to the field by a welded 6-in. line was chosen. Construction of the plant proper began in August, 1928, and by April, 1929, it was producing helium.

The plant buildings and all of the special equipment dealing with helium extraction were designed by Bureau of Mines engineers. Practically all of the special equipment now in use was fabricated at the Bureau's Amarillo helium plant, where a very complete machine shop, welding shop, and auxiliary equipment are maintained. Special recorders were constructed to indicate continuously the purity of the crude helium and that of the final product as delivered to the shipping containers. Other recorders analyze the nitrogen that is extracted in small quantities from the natural gas for use in the closed nitrogen-refrigeration cycle. Potentiometers are used for indicating the tem-

TABLE 1 GOVERNMENT HELIUM PRODUCTION AND COSTS, APRIL, 1921, TO DECEMBER, 1934

| Period | Production[a] cu ft | Gross operating cost (expenditures in operation and maintenance)[b] Total | Average per M cu ft produced | Return from sale of residue gas | Net operating cost (gross operating cost less return from sale of residue gas)[b] Total | Average per M cu ft produced |
|---|---|---|---|---|---|---|
| Fort Worth plant:[c] | | | | | | |
| Under jurisdiction of Navy Department: | | | | | | |
| April to June, 1921 | 260,520 | $126,694.05 | $486.31 | ..... | . | .... |
| July to December, 1921 | 1,841,000 | 320,859.73 | 174.28 | ..... | ..... | ... |
| October, 1922, to June, 1923[d] | 4,069,940 | 489,299.70 | 120.22 | ..... | ..... | ..... |
| July, 1923, to June, 1924 | 8,204,665 | 636,438.38 | 77.57 | ..... | ..... | ..... |
| July, 1924, to June, 1925 | 9,418,363 | 451,084.58 | 47.89 | ..... | ..... | ..... |
| | 23,794,488 | 2,024,376.44 | 85.08 | ..... | ..... | ..... |
| Under jurisdiction of Bureau of Mines: | | | | | | |
| July, 1925, to June, 1926 | 9,355,623 | 318,446.40 | 34.04 | ..... | ..... | ..... |
| July, 1926, to June, 1927 | 6,330,056 | 277,384.70 | 43.82 | ..... | ..... | ..... |
| July, 1927, to June, 1928 | 6,687,834 | 274,210.54 | 41.00 | ..... | ..... | ..... |
| July, 1928, to Jan. 10, 1929 | 2,638,894 | 121,440.65 | 46.02 | ..... | ..... | ..... |
| | 25,012,407 | 991,482.29 | 39.64 | ..... | ..... | ..... |
| Amarillo plant:[e] | | | | | | |
| Under jurisdiction of Bureau of Mines: | | | | | | |
| April to June, 1929 | 844,900 | 27,833.16 | 32.94 | $ 2,645.32 | $25,187.84 | $29.81 |
| July, 1929, to June, 1930 | 9,805,600 | 140,146.75 | 14.30 | 30,445.43 | 109,701.32 | 11.19 |
| July, 1930, to June, 1931 | 11,362,730 | 150,190.53 | 13.22 | 32,510.24 | 117,680.29 | 10.36 |
| July, 1931, to June, 1932 | 15,171,680 | 148,545.26 | 9.79 | 40,862.43 | 107,682.83 | 7.10 |
| July, 1932, to June, 1933 | 14,749,960 | 151,165.51 | 10.25 | 37,661.70 | 113,503.81 | 7.70 |
| July, 1933, to June, 1934 | 6,534,270 | 63,528.33 | 9.72 | 17,585.94 | 45,942.39 | 7.03 |
| July to December, 1934 | 6,391,270 | 54,954.24 | 8.60 | 16,762.42 | 38,191.82 | [f]5.98 |
| | 64,860,410 | 736,363.78 | 11.35 | 178,473.48 | 557,890.30 | 8.60 |

[a] Production from the Fort Worth plant represents volume of airship gas produced, which had an average helium purity of 93 to 94 per cent under Navy jurisdiction and about 95 per cent under Bureau of Mines jurisdiction. Production from the Amarillo plant represents actual helium in the airship gas of better than 98 per cent purity produced by that plant. Therefore, the advantage of the Amarillo plant from standpoint of cost is about 5 per cent greater than a direct comparison of the figures indicates.

[b] Gross operating costs for the Fort Worth plant represent expenditures in operating and maintaining the plant, including current expenditures for natural gas. The Government did not own the gas field that supplied the Fort Worth plant, so there was no return from sale of residue. Gross operating cost for the Amarillo plant represents expenditure in operating and maintaining both the plant and the Government-owned gas properties. This gross operating cost at Amarillo is a measure of the amount that must be available to the Bureau of Mines for current expenditure. Returns from sale of residue gas, in excess of its cost, must be deposited to credit of miscellaneous receipts of the Treasury and therefore are not available for expenditure by the Bureau. As the net operating cost is computed by subtracting current returns from current expenditures, it is a measure of the net withdrawal of funds from the Treasury for operation and maintenance.

[c] Costs at the Fort Worth plant are based on compilations by the Bureau of Efficiency from records of the Navy Department and the Bureau of Mines. (Report of Bureau of Efficiency in hearing on Amarillo helium plant before the Committee on Mines and Mining, House of Representatives, 71st Congress, second session, p. 210.) The costs do not include depreciation or depletion, and those for period of Navy jurisdiction do not include cost of Washington administration.

[d] Plant closed in 1922 from January to September, inclusive, because of lack of funds.

[e] Compiled from Bureau of Mines records. The costs do not include depreciation or depletion.

[f] Unit costs for year 1934 abnormally low because of Government pay cuts, furlough of employees, and reduction of plant crew to the minimum required to man plant and gas field for intermittent operation. In normal times considerably higher unit costs for a like volume of production may be expected. Average costs for entire fiscal year 1935 probably will exceed those for the first 6 months.

perature in various parts of the equipment; high-pressure liquid-level gages on the U-tube principle have been designed and constructed for determining the level of low-temperature liquids in various containers under pressures of approximately 3000 lb per sq in. A brief description of the operation of the plant is as follows:

The first step in the process is the removal of the small amount of carbon dioxide which the natural gas contains. The Bureau has carried on extensive research looking toward a more satisfactory method of performing this operation, but as yet nothing that has decided advantages over the system developed by the Navy Department at the old Fort Worth plant has been found. Carbon dioxide is removed by scrubbing the incoming gas with a 7-per cent solution of sodium hydroxide and the resulting sodium carbonate is reconverted to hydroxide by the use of lime. The gas leaves the carbon-dioxide scrubbers at approximately pipe-line pressure of about 650 lb per sq in. By means of heat exchangers the gas is cooled progressively to approximately 300 deg below zero. At that temperature and under the pressure used in this part of the apparatus, virtually all of the constituents of the natural gas except helium and a small amount of nitrogen become liquid. The helium is withdrawn from the top part of the apparatus and the liquid from the bottom. The liquids are returned through the exchanger countercurrent to the incoming stream, and the gas resulting from their evaporation is sold to the local gas company. A complete trip through the entire apparatus for any given cubic foot of gas takes place in less than a minute.

The heat leaks in the system are made up primarily by means of the expansion-engine cycle. Nitrogen that is separated from the natural gas is compressed and sent through an expansion engine. The engine runs an electrical generator to remove heat energy from the nitrogen. As a result, the engine exhaust is lowered and the exhaust is used to cool the high-pressure nitrogen flowing to the engine. This continues progressively until a sufficiently low exhaust temperature has been reached. This nitrogen-refrigeration cycle is entirely separate from the gas cycle. It would, of course, be easier to use air as a refrigerating medium than to use nitrogen; however, air or the resulting liquid oxygen from it might possibly come in contact with liquid methane through accidental leaks. Liquid oxygen and liquid methane form a dangerous explosive, which explodes with little or no provocation and certainly no warning. In violence such a mixture is approximately equal to nitroglycerine.

The minimum purity of helium shipped from the Amarillo helium plant is 98.2 per cent. This helium is produced in two

steps. In the first operation a crude helium is obtained having a composition of about 50 per cent nitrogen and 50 per cent helium. High-purity helium can be produced in one cycle, but the resulting loss of helium due to the solubility effect warrants the use of two separate cycles. By producing the helium in two operations, which are carried on simultaneously, the final product of 98.2 per cent helium leaves the unit at high pressure and can be discharged directly into tank cars or other shipping devices, thus avoiding compression of the pure helium, which is not only difficult but costly.

From the standpoint of what research can do for an industry, it is interesting to note the comparison between the performance of the old Fort Worth plant and the Amarillo helium plant. However, it should be borne in mind that the operation of the Fort Worth plant was very creditable when one considers the lack of information and data at the time of its design and construction. During the first real operating period of the Fort Worth plant, the cost of operation was approximately $175 per thousand cubic feet of the helium produced. The cost was reduced rapidly, until by June, 1925, it had reached a yearly average of about $47 per thousand cubic feet. The best yearly figure for the Fort Worth production, that of July, 1925, to June, 1926, was about $34 per thousand cubic feet. A total of approximately 48,000,000 cu ft of helium was produced during the operating life of the Fort Worth plant.

The Amarillo helium plant began operations in April, 1929. To date it has produced more than 70,000,000 cubic feet of helium at an average gross operating cost of approximately $12 per thousand cubic feet; however, it has returned to the Treasury approximately $200,000 from the sale of the residue gas, giving a net average operating cost of about $9 per thousand cubic feet of helium.

During several months of high production, the operating cost on the net basis was below $5 per thousand cubic feet, and this figure could, in all probability, be maintained if the demand for helium allowed operating under full capacity. Table 1, taken from the last report of the Bureau of Mines as given in the Minerals Yearbook (current figures in detail have not been released), will give a more exact account of the cost of operation. It might be noted that the difference in cost of operating the Amarillo helium plant from the standpoint of money withdrawn from the Treasury as compared with the production of an equal quantity of helium at the lowest cost attained at Fort Worth, namely $34 per thousand, indicates that the Amarillo helium plant has saved the Government to date something more than $1,700,000. Since the plant itself represents an investment of between $700,000 and $800,000, it is evident that approximately a million dollars has been saved, which in effect could apply on the Government investment in its natural-gas properties. From the standpoint of helium production, it is evident that applied research has paid.

# Modified I.S.A. Orifice With Free Discharge

By M. P. O'BRIEN[1] and R. G. FOLSOM,[2] BERKELEY, CALIF.

**The authors present results obtained with a modified type of International Standard Orifice as used for metering pump discharges. The annular slot for pressure measurement of the standard I.S.A. orifice is replaced in the modified orifice by a series of slots at the pipe wall and flush with the orifice plate. The results given in the paper include coefficients obtained with sharp-edged and rounded-edged orifices with various orifice-pipe diameter ratios and with various approach lengths of pipe. The coefficients obtained by the authors with the modified I.S.A. orifice are compared with coefficients of the standard I.S.A. orifice.**

FIELD CONDITIONS do not permit universal use of any one metering method for measuring pump discharges and therefore a relatively large number are in common use. A device which has been used widely for metering irrigation deliveries is a sharp-edged orifice attached to the end of the discharge pipe. This method is probably as accurate as any of the others commonly used for field testing of pumps, but suffers from the fact that the discharge coefficient has not been accurately measured. The present paper deals with a particular arrangement of this type of free discharge orifice, which will be referred to here as the modified International Standard Orifice.

The International Standard Orifice[3] is normally operated submerged as a diaphragm or pipe-line orifice. The pressure connections consist of annular slots at the same radius as the wall of the pipe and flush against the orifice plate. The entire assembly of orifice-plate and pressure connections forms a unit which is inserted in the pipe line. In addition to the advantage of having the pressure connections made integral with the orifice, the I.S.A. orifice has been carefully investigated and the conditions of use, the tolerances in manufacture, and the accuracy have been specified in detail. The I.S.A. standards specify the coefficients for free discharge of a liquid into a gas but these coefficients were not based on adequate experimental data and the purpose of the work reported here was to supply this information.

The modified form of the I.S.A. orifice is shown in Fig. 1. The annular slot for pressure measurement of the standard I.S.A. orifice was replaced by a series of slots at the pipe wall and flush with the orifice plate, as shown in Fig. 2. Only one pressure connection is necessary with free discharge. The orifice was sharp-edged without rounding or burrs and the upstream face was smooth. The orifice plate was made of machined stainless steel and the holder was of cast aluminum.

## Equipment and Procedure

*Setup With 8-In. Pipe.* The orifice in the 8-in. pipe was attached to the end of a horizontal section of new standard black wrought pipe of 8 in. nominal diameter and actual average internal diameter of 8.10 in. The arrangement of the piping and the manometer are shown in Fig. 1. The approach length $L$ was varied from 4 in. to 21 ft. The discharge was measured volumetrically in a calibrated tank having a capacity of 400 cu ft. The minimum collection period was 90 sec.

*Setup With 6-In. Pipe.* The equipment and arrangement for the 6-in. pipe were similar to the 8-inch apparatus. Series of tests were made using 20 and 23 diameters of old corroded pipe having an average internal diameter of 6.13 in. and 50 diameters of new standard black wrought pipe of 6.067 in. average internal diameter. The maximum and minimum diameters in the old pipe were 6.188 and 6.115 in., respectively.

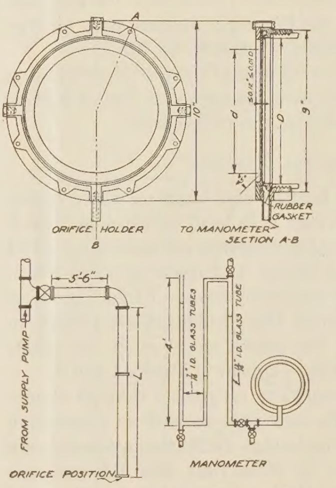


Fig. 1 The Modified I.S.A. Orifice and Piping Arrangement Used in Tests

All measurements of diameters were made with machinists' inside micrometers. A minimum of ten determinations on different equally spaced diameters were averaged to obtain the sizes given. Pipe diameters were measured at the section 1 in. upstream from the orifice.

## Discharge Equation

The selection of a discharge equation is largely a matter of convenience in application. In the case of the free-discharge orifice, there appears to be no advantage in including the velocity of approach explicitly. For the I.S.A. orifice, the basic equation has been assumed as

$$Q = CA\sqrt{2gH} \qquad [1]$$

where, in English units, $Q$ = rate of discharge, cfs; $C$ = a dimensionless coefficient; $A$ = orifice area, sq ft; $g$ = weight per unit mass = 32.2 ft per sec per sec; and $H$ = head of water measured above the center of the horizontal pipe line, ft. The coefficients mentioned in this paper have all been computed by means of Equation [1].

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[2] Instructor, Department of Mechanical Engineering, University of California. Jun. A.S.M.E. Mr. Folsom received the degrees of B.S., M.S., and Ph.D. from the California Institute of Technology in 1928, 1929, and 1932, respectively. Since his graduation in 1928 he has worked for one year as part-time research assistant at Riverside Cement Company, Riverside, California; four years as teaching fellow at the California Institute of Technology; and one year as engineer for the Water Department of the City of Pasadena, California. He has been at the University of California since 1933.

[3] "Regeln für die Durchflussmessung mit genormten Düsen und Blenden," Deutscher Industrie Normen (DIN), No. 1952, third Auflage, V.D.I. Verlag, Berlin, 1935. The organization from which the orifices and nozzles derive their name is the International Federation of National Standardizing Associations.

Contributed by the Special Research Committee on Fluid Meters and presented at the Annual Meeting of The American Society of Mechanical Engineers, held in New York, N. Y., November 30 to December 4, 1936.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until February 10, 1937, for publication at a later date. Discussion received after the closing date will be returned.

The coefficient $C$ is a function of the diameter or area ratio, the Reynolds number, the roughness and length of the approach pipe, and, to a minor extent, of a generalized parameter representing the surface tension. In the range of heads and orifice sizes which might be used in the field, the effect of surface tension is negligible. As applied to the measurement of water at normal temperature, the Reynolds number depends largely on the head which may be demonstrated as follows:

Fig. 2 Slots in the Pipe Wall Flush With the Modified I.S.A. Orifice

The Reynolds number

$$Re = V\left(\sqrt{\frac{4A}{\pi}}\Big/\nu\right)$$

and since

$$V = Q/A = C\sqrt{2gH}$$

$$Re = C\left(\sqrt{\frac{8g\,AH}{\pi}}\Big/\nu\right)$$

where the kinematic viscosity $\nu$ is nearly constant, $A$ is a fixed dimension, and $C$ is itself a function of the Reynolds number and the area ratio. Therefore, for any particular orifice, the effect of the Reynolds number may be represented by plotting the coefficient against the head.

The I.S.A. standards use $A$ as the area of the pipe in computing Reynolds' number while the new standards of the A.S.M.E. Fluid Meters Committee uses $A$ as the orifice area.

## Experimental Results

Except for short approach lengths and large diameter ratios, the orifice coefficient does not vary with head in the range tested. In Figs. 3 and 4, the measured coefficients are shown for all of the combinations of approach length and diameter ratio which showed a variation in coefficient with head. Excluding those conditions giving such variations, the coefficients are plotted in Fig. 5, for comparison with the values specified in the I.S.A. standards. The summarized data appear in Tables 1 and 2.

With the approach length equal to or greater than 20 diameters of pipe, the coefficients for new pipe showed no progressive trend with head, and the deviations from the average were small. The mean deviations of these runs were 0.5 per cent or less for all combinations. With an approach length equal to 8 pipe diameters, the coefficient showed no trend with head below a diameter ratio of 0.6, but above this ratio, the coefficient decreased with increasing head. The shortest approach pipe consisted simply of a short nipple between the orifice holder and the flange of the elbow. With this arrangement the coefficient decreased with increasing head at all diameter ratios. With the shortest approach length, the jet was rough and rotated at such a high angular velocity as to cause it to break apart. Fig. 6 shows this condition while Fig. 7 shows a normal jet.

The coefficients with the old corroded 6-in. pipe were high as compared with the other values. The increase in coefficient was only about one half the value given for pipe roughness in the I.S.A. standards. With rough pipe, the tolerance of minimum Reynolds' number for constant coefficient and required length of straight pipe is apparently higher than with smooth pipe.

The Reynolds number is a measure of the importance of friction, a low Reynolds number corresponding to a larger relative friction. From this viewpoint, it would appear that the decreasing coefficients observed might be ascribed to the effect of Reynolds' number. However, it is generally observed that as the turbulence increases because of such factors as roughness and obstructions, the coefficients of discharge and friction

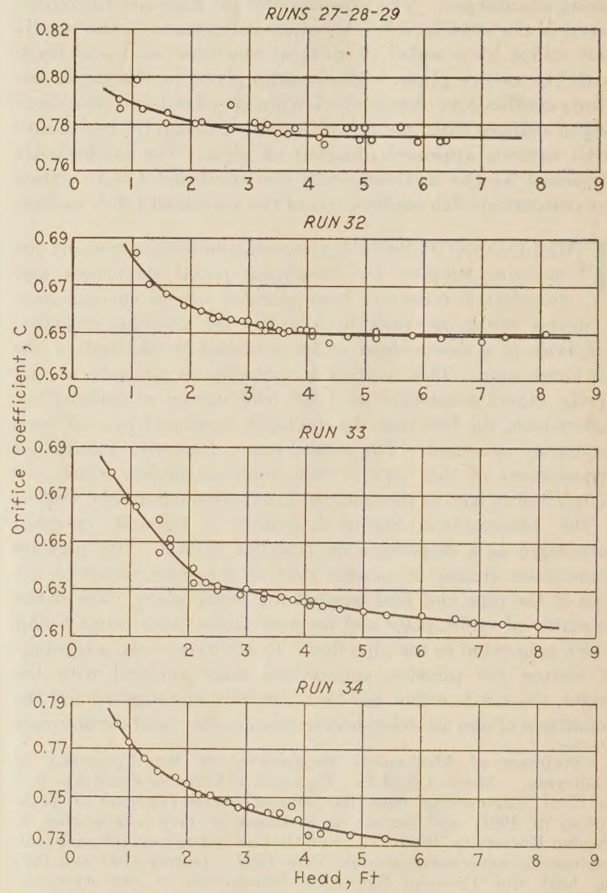


Fig. 3 Variations in Orifice Coefficient With Head for 8-In. Pipe

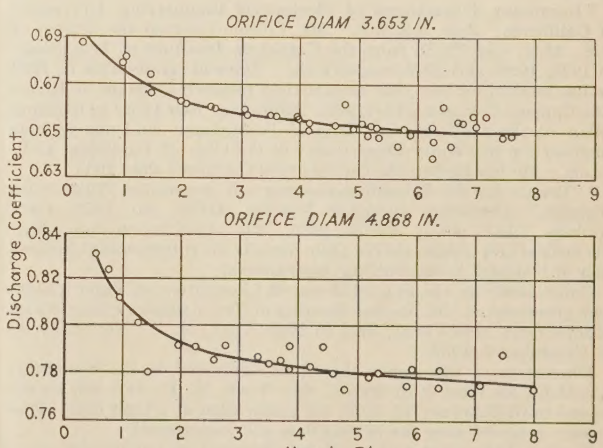


Fig. 4 Variation in Discharge Coefficient With Head for 6-In. Pipe

TABLE 1 SUMMARY OF EXPERIMENTAL RESULTS FOR NORMAL SHARP-EDGED ORIFICES AND NONSTANDARD ORIFICES WITH NEW STANDARD 8-IN. PIPE

| | Run no. | Ratio Diameter | Ratio Area | Coefficient Average | Coefficient Deviations, per cent Mean | Coefficient Deviations, per cent Max | Length of approach pipe, diameters | Test quantities Minimum Head, ft | Test quantities Minimum Discharge sec-ft | Test quantities Maximum Head, ft | Test quantities Maximum Discharge sec-ft | No. of runs |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Normal sharp-edged orifices | 1–11A | 0.797 | 0.636 | 0.759 | 0.5 | 2.6 | 31 | 0.57 | 1.05 | 6.75 | 3.60 | 59 |
| | 12–13 | 0.598 | 0.358 | 0.646 | 0.2 | 0.8 | 31 | 0.62 | 0.52 | 8.09 | 1.89 | 36 |
| | 16–18 | 0.401 | 0.161 | 0.610 | 0.2 | 0.7 | 31 | 2.75 | 0.45 | 7.50 | 0.77 | 28 |
| | 19–20 | 0.401 | 0.161 | 0.612 | 0.1 | 0.3 | 8 | 1.81 | 0.38 | 7.79 | 0.79 | 19 |
| | 24–26 | 0.797 | 0.636 | 0.766 | 0.3 | 0.8 | 20 | 1.41 | 1.66 | 6.74 | 3.64 | 26 |
| | 27–29 | 0.797 | 0.636 | 0.775[b] | 0.3[b] | 0.5[b] | 8 | 4.19[b] | 2.89 | 6.37 | 3.57 | 30 |
| | 35 | 0.401 | 0.161 | 0.612 | 0.4 | 0.9 | 20 | 1.15 | 0.30 | 7.63 | 0.78 | 8 |
| | 30–31 | 0.598 | 0.358 | 0.655 | 0.3 | 0.7 | 8 | 0.91 | 0.64 | 8.24 | 1.94 | 21 |
| | 32 | 0.598 | 0.358 | 0.649[b] | 0.2[b] | 0.5[b] | 1 | 4.16[b] | 1.36 | 8.13 | 1.91 | 28 |
| | 33 | 0.401 | 0.161 | ...[a] | ... | ... | 1 | 0.74[a] | ... | 8.02 | ... | 30 |
| | 34 | 0.797 | 0.636 | ...[a] | ... | ... | 1 | 0.77[a] | ... | 5.45 | ... | 20 |
| | Spec. | 0.471 | 0.222 | 0.619 | ... | ... | 20 | ... | ... | ... | ... | 2 |
| | Spec. | 0.662 | 0.437 | 0.672 | ... | ... | 20 | ... | ... | ... | ... | 2 |
| Nonstandard orifices | 14–15[c] | 0.401 | 0.161 | 0.615 | 0.2 | 0.3 | 31 | 2.69 | 0.47 | 4.84 | 0.63 | 6 |
| | | | | 0.718 | 0.2 | 0.4 | | 4.96 | 0.74 | 7.56 | 0.91 | 8 |
| | 21–22[c] | 0.354 | 0.125 | 0.613 | 0.2 | 0.5 | 20 | 1.53 | 0.27 | 8.25 | 0.63 | 20 |
| | 23[c] | 0.545 | 0.297 | 0.638 | 0.3 | 0.7 | 20 | 1.12 | 0.58 | 8.29 | 1.57 | 11 |
| | 36[d] | 0.401 | 0.161 | 0.613 | 0.2 | 0.6 | 20 | 0.74 | 0.24 | 8.16 | 0.81 | 28 |
| | Spec.[c] | 0.787 | 0.618 | 0.763 | ... | ... | 20 | ... | ... | ... | ... | 2 |

[a] Coefficient decreased with increasing head in range of experiment.
[b] Average for heads over 4 ft, see Fig. 3.
[c] Orifice rounded.
[d] Orifice notched.

TABLE 2 SUMMARY OF EXPERIMENTAL RESULTS FOR NORMAL SHARP-EDGED ORIFICES WITH STANDARD 6-IN. PIPE

| Condition of pipe | Ratio Diameter | Ratio Area | Coefficient Average | Coefficient Deviations, per cent Mean | Coefficient Deviations, per cent Max | Length of approach pipe, diameter | Test quantities Minimum Head, ft | Test quantities Minimum Discharge, sec-ft | Test quantities Maximum Head, ft | Test quantities Maximum Discharge, sec-ft | No. of runs |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Rough | 0.596 | 0.355 | 0.651[a] | ... | ... | 19.6 | 4.14 | 0.86 | 7.60 | 1.04 | 31 |
| Rough | 0.794 | 0.631 | 0.777[a] | ... | ... | 19.6 | 4.57 | 1.73 | 7.50 | 2.21 | 12 |
| Rough | 0.596 | 0.355 | 0.650 | 0.4 | 1.2 | 22.6 | 0.37 | 0.23 | 7.89 | 1.07 | 31 |
| Rough | 0.794 | 0.631 | 0.778 | 0.4 | 1.0 | 22.6 | 0.37 | 0.49 | 7.60 | 2.22 | 23 |
| New | 0.601 | 0.362 | 0.649 | 0.2 | 0.4 | 50.0 | 1.02 | 0.38 | 8.02 | 1.07 | 15 |
| New | 0.802 | 0.643 | 0.774 | 0.05 | 0.1 | 50.0 | 1.69 | 1.05 | 6.66 | 2.07 | 8 |

[a] Average coefficients for heads greater than 4 ft.

become constant at lower Reynolds numbers than with undisturbed flow. The opposite effect is evident here since the shorter approach length evidently causes more turbulence, and yet the coefficient becomes constant at a higher head or Reynolds number than with the longer approach lengths. The question has some practical importance in the application of the data to other pipe sizes, if such application is to be made, but it is strongly recommended that short approach lengths and old corroded pipes be avoided.

As indicated in Table 1, the tests on one orifice for the 8-in. pipe showed an abrupt break in the coefficient from 0.615 to 0.718 at a head of 4.9 ft. The orifice diameter was 3.255 in. and the approach length was 31 diameters. After this orifice had been machined initially, a slight burr could be felt and the machinist, acting without instructions, took a beveled cut to smooth it off. The cut was not noticed until the peculiarities of the coefficient appeared. The series was repeated by another observer with increasing and decreasing heads with the same results. Measurements of the jet with calipers showed the diameter of the vena contracta increased in agreement with the change in coefficient. This sensitivity to the condition of the orifice edge suggested an experiment on the effect of such damage to the edge as might occur in handling. An orifice with the same diameter but with a sharp edge was tested with a 20-diameter length of approach pipe. The edge was then dented with a cold chisel at 46 points. The average coefficient before and after the edge was damaged was 0.615. Fig. 8 shows the jet obtained with the damaged orifice.

The fact that the coefficient for this orifice at lower heads agrees with the general trend suggests that the jet broke clear from the upstream face at low heads but that a slight decrease in the radius of curvature let the surface of the jet touch the beveled face. As soon as it touched, the outer filaments were both slowed up and deflected and the abrupt increase in the area of the jet resulted. The effect would certainly be unpredictable in magnitude and the conclusion to be drawn is

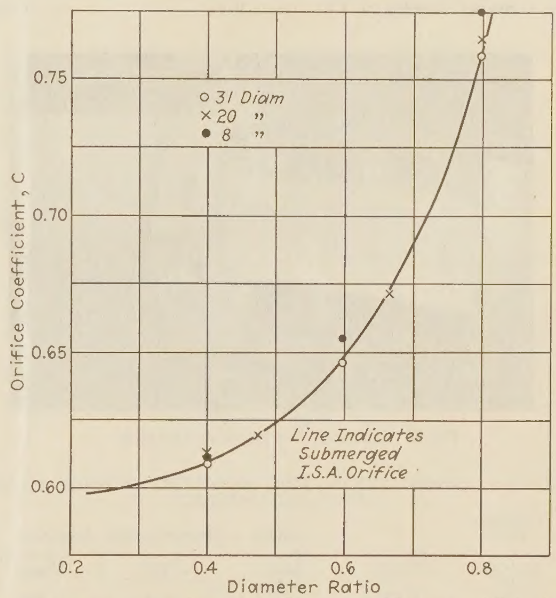


FIG. 5 COEFFICIENTS OBTAINED WITH THE MODIFIED I.S.A. ORIFICE COMPARED WITH NORMAL I.S.A. ORIFICE COEFFICIENTS

that the orifice edge should be as sharp as can be produced by careful machining.

The second set of data in Table 1 includes experiments on a number of other orifices with slightly rounded edges, none of which show a discontinuity in the coefficient. The exact shape of the edge could not be measured. The coefficients all lie slightly above those obtained with sharp-edged orifices by amounts which might well be in proportion to the increase in effective area of the aperture. Fig. 9 shows a comparison of these coefficients with the normal values.

### Recommended Coefficients

The diameters of the pipes used are in the lower portion of the range for which this type of orifice is suitable. The fact that the coefficient is constant in the recommended range indicates that the same coefficients may be employed for larger pipe sizes. Diameter ratios in excess of the maximum tested are not recommended. The measured coefficients differ from the coefficients recommended for the I.S.A. orifice discharging submerged by amounts which are less than the average deviations of the data.

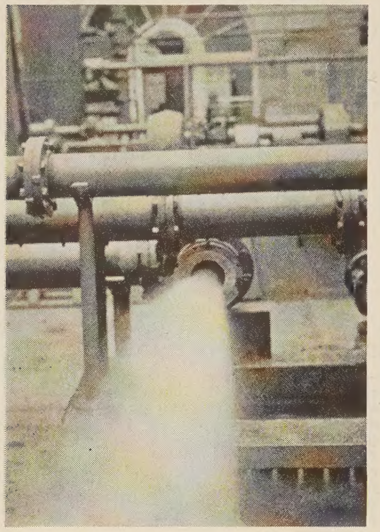

Fig. 6 Jet From Orifice at the End of a Short Approach Length of Pipe

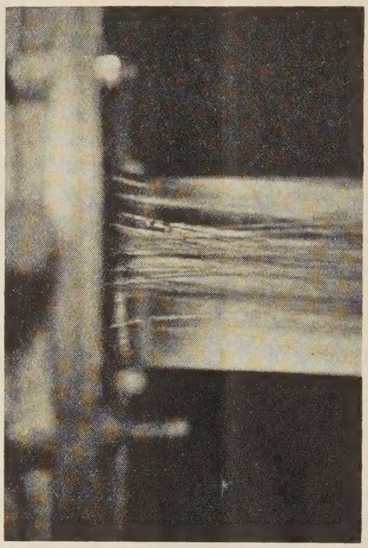

Fig. 8 Jet From a Damaged Orifice

On the basis of this agreement with the I.S.A. orifice data and the internal evidence of the data presented here, extrapolation of the data to larger pipe sizes is believed to be justified. The recommended coefficients for new pipe of diameters 6 in. and greater are given in Table 3.

An approximate formula representing the measured coefficients up to a diameter ratio of 0.8 is

$$C = 0.60 + 0.40\left(\frac{d}{D}\right)^4 \ldots\ldots\ldots [2]$$

This formula represents the experimental data and the coefficients of the I.S.A. orifice within 1 per cent up to a diameter ratio of 0.8 with approach lengths of smooth pipe equal to 20 diameters and greater.

The coefficients given for approach lengths less than 20 diameters apply only to the piping arrangement used for these tests. Other experiments show that the piping arrangement above the last elbow exerts a marked but unpredictable influence on the coefficient.

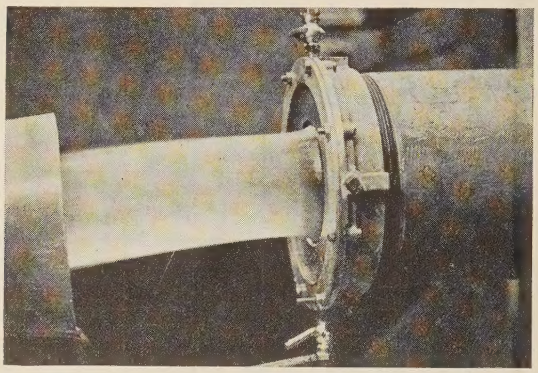

Fig. 7 Normal Jet From an Orifice

TABLE 3 COEFFICIENTS FOR MODIFIED INTERNATIONAL STANDARD ORIFICES[a]

| Diameter ratio | Length of approach pipe, diameters 31 | 20 | 8 | $L < D$ |
|---|---|---|---|---|
| 0.401 | 0.610 | 0.612 | 0.612 | Varied |
| 0.471 | ... | 0.619 | ... | ... |
| 0.598 | 0.646 | ... | 0.655 | 0.649 |
| 0.662 | ... | 0.672 | ... | ... |
| 0.797 | 0.759 | 0.766 | 0.775 | Varied |

[a] Recommended coefficients, given to the left of the line in the table, are for a minimum head of 12 in. for 6-in. pipe and larger. The probable error of a single measurement is less than 1 per cent.

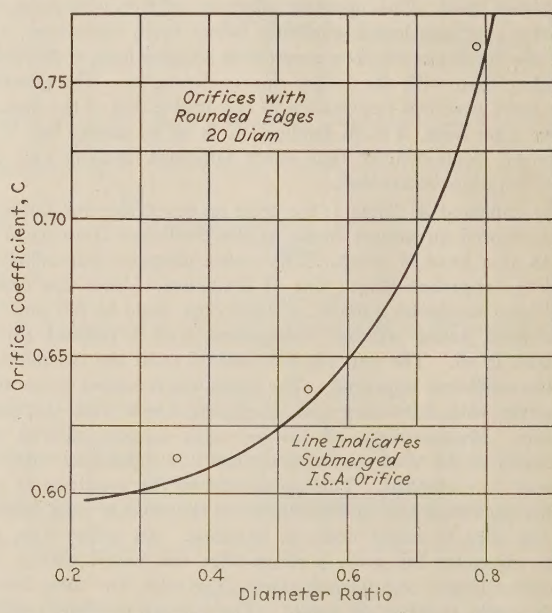


Fig. 9 Coefficients Obtained With a Modified I.S.A. Orifice Having a Slightly Rounded Edge Compared With I.S.A. Orifice Coefficients